Online Appendix to
"Uncertainty, Financial Frictions, And Nominal Rigidities: A Quantitative Investigation"

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This Online Appendix includes additional details and results to Cesa-Bianchi and Fernandez-Corugedo (2017).

Section A reports the equilibrium conditions of the model. Section B shows how we compute the steady state of the model. Section C explains the methodology we use to compute the impulse response functions reported in the main text. Section D reports the results of a comparative steady-state exercise that sheds light on the transmission mechanism of micro uncertainty shocks. Section E reports some additional impulse response functions that complement the analysis in the main text of the paper.

A Equilibrium

Define $q_t \equiv Q_t/P_t$, $nw_t \equiv NW_t/P_t$, $z_t \equiv Z_t/P_t$. For a given path for the exogenous processes, a recursive (imperfectly) competitive equilibrium of the model is a sequence of allocations for the endogenous variables that solves the following system of equations.

Euler equation of households:

$$U_{c,t} = \beta(1 + R^n_t)\mathbb{E}_t \left[\frac{U_{c,t+1}}{\pi_{t+1}}\right]. \tag{1}$$

Labor supply:

$$mc_t Y_{n,t} = -\frac{U_{n,t}}{U_{c,t}}. \tag{2}$$

Marginal product of capital:

$$mc_t Y_{k,t} = z_t. \tag{3}$$

Price of capital:

$$q_t = \left[1 - \phi_k \left(\frac{I_t}{K_t} - \delta\right)\right]^{-1}. \tag{4}$$

Zero profit condition:

$$y^k_{t+1}K_{t+1}\Big(\Gamma(\bar{\omega}_{t+1})-\mu G(\bar{\omega}_{t+1})\Big)=(1+R^n_t)(q_tK_{t+1}-nw_{t+1}). \tag{5}$$

NK Phillips curve:

$$(\pi_t-\pi)\,\pi_t=\beta\mathbb{E}_t\left\{\frac{U_{c,t+1}}{U_{c,t}}\left(\pi_{t+1}-\pi\right)\pi_{t+1}\right\}+Y_t\frac{\varepsilon}{\omega_p}\left(mc_t-\frac{\varepsilon-1}{\varepsilon}\right) \tag{6}$$

Net worth law of motion:

$$nw_{t+1}=\gamma y^k_{t+1}K_{t+1}\left(1-\Gamma(\bar{\omega}_{t+1})\right). \tag{7}$$

Entrepreneurs real consumption:

$$C^e_t=(1-\gamma)\left(1-\Gamma(\bar{\omega}_{t+1})\right)y^k_tK_t. \tag{8}$$

Aggregate resource constraint:

$$A_tF(K_t,N_t)=C_t+C^e_t+I_t+\frac{\omega_p}{2}\left(\pi_t-\pi\right)^2+\mu G(\bar{\omega})y^k_tK_t. \tag{9}$$

Accumulation of aggregate capital:

$$K_{t+1}=(1-\delta)K_t+I_t-\frac{\phi_k}{2}\left(\frac{I_t}{K_t}-\delta\right)^2K_t. \tag{10}$$

Monetary policy:

$$\frac{1+R^n_t}{1+R^n}=\left(\frac{1+R^n_{t-1}}{1+R^n}\right)^{\phi^r}\left(\frac{1+\pi^n_t}{1+\pi}\right)^{(1-\phi^r)\phi^\pi}\left(\frac{1+Y_t}{1+Y_{t-1}}\right)^{(1-\phi^r)\phi^y}. \tag{11}$$

Definition of real income from holding one unit of finished capital:

$$y^k_t=z_t+q_t\left[1-\delta-\frac{\phi_k}{2}\left(\frac{I_t}{K_t}-\delta\right)^2+\phi_k\left(\frac{I_t}{K_t}-\delta\right)\frac{I_t}{K_t}\right]. \tag{12}$$

$$y^k_{t+1}=\frac{(1+R^k_{t+1})q_t}{\pi_{t+1}}. \tag{13}$$

Optimal contract:

$$\frac{1+R^k_{t+1}}{1+R^n_t}=\psi_t. \tag{14}$$

where:

$$\psi_t=\left(\frac{\Big(1-\Gamma(\bar{\omega}^j_{t+1})\Big)\Big(\Gamma'(\bar{\omega}^j_{t+1})-\mu G'(\bar{\omega}^j_{t+1})\Big)}{\Gamma'(\bar{\omega}^j_{t+1})}+\Big(\Gamma(\bar{\omega}^j_{t+1})-\mu G(\bar{\omega}^j_{t+1})\Big)\right)^{-1}. \tag{15}$$

B Steady State

To compute the steady state of the model, we take an approach similar to Faia and Monacelli (2007). First, notice that some value steady-state values can be pinned down simply by the calibrated parameters. For example, from the Euler equation of consumption notice that:

$$1 + R^n = \pi/\beta.$$

From the New Keynesian Phillips curve:

$$mc = \frac{\varepsilon - 1}{\varepsilon}.$$

From the price of capital equation:

$$q = 1.$$

Second, the entrepreneurial problem has to be solved to compute the cut-off value of the idiosyncratic productivity. In order to do that, notice that it is possible to compute the net worth to capital ratio from both the zero profit condition of banks:

$$\mathcal{NK}_1 = \frac{nw}{K} = 1 - \frac{y^k}{1 + R^n}\Big(\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})\Big)$$

and from the law of motion of net worth:

$$\mathcal{NK}_2 = \frac{nw}{K} = \gamma y^k \left(1 - \Gamma(\bar{\omega})\right),$$

where remember that $y^k = (1 + R^k)/\pi$. By guessing an initial value for $\bar{\omega}$ we can compute R^k from the efficiency conditions associated with the optimal contract. With a simple algorithm in MatLab, it is then possible to modify $\bar{\omega}$ until the following condition $\mathcal{NK}_1 = \mathcal{NK}_2$ is satisfied. Once the steady-state level of $\bar{\omega}$ is determined, R^k, y^k, ψ, $\Gamma(\bar{\omega})$, and $G(\bar{\omega})$ are also determined.

To compute the steady-state value of the remaining variables, notice that from the definition of the nominal income from holding one unit of capital:

$$z = y^k - 1 + \delta.$$

Then, we can compute the following ratios from the production function:

$$\begin{aligned} \frac{Y}{K} &= \frac{z}{\alpha \cdot mc}, \\ \frac{K}{N} &= \left(\frac{Y}{K}\right)^{\frac{1}{\alpha - 1}}, \end{aligned}$$

from the law of motion of capital:

$$\frac{I}{K} = \delta,$$

and from the aggregate resource constraint:

$$\frac{C}{K} = \frac{Y}{K} - \frac{I}{K} - \mu G(\bar{\omega}) y^k.$$

Finally, by fixing the steady-state level of hours $N = 1/3$ it is possible to solve the above equations and easily compute the remaining endogenous variables of the model.

C Impulse Response Calculation

To compute the impulse responses reported in the paper we use a two steps procedure. As noted by Fernandez-Villaverde et al. (2011), the higher order approximation makes the simulated paths of states and controls in the model move away from their steady-state values. This is actually one of the results of Schmitt-Grohe and Uribe (2004): in a first-order approximation of the model, the expected value of any variable coincides with its value in the non-stochastic steady state, while in a second-order approximation of the model, the expected value of any variable differs from its deterministic steady-state value only by a constant.

In a third order approximation, the expected value of the variable will also depend on the variance of the shocks in the economy. Therefore, it is more informative to compute impulse responses as percentage deviations from their mean, rather than their steady state.

However, a well-known flaw of higher-order perturbations is that when the approximated decision rules are used to produce simulated time series from the model, the simulated data often display an explosive behavior. We address the problem of explosive paths of simulated data by applying the pruning procedure by Kim et al. (2008).[1]

In the first step we simulate the model and compute the mean of the state and control variables. In particular we:

1. Draw a series of random shocks $\varepsilon_t = (\varepsilon_t^A, \varepsilon_t^W, \varepsilon_t^S)$ for T periods ($T = 4000$)
2. Starting from the steady state, perform simulation of the model using ε_t and get Y_t (i.e., the simulated data)
3. Discard the first half of observations as a burn in, and compute the ergodic mean of Y over the last $0.5 \cdot T$ periods:

$$Y_0 = \frac{\sum_{0.5T+1}^{T} Y_t}{0.5T}$$

In the second step, we compute impulse responses. For example, for the macro uncertainty shock (ε_t^W) we:

1. Draw a series of random shocks ε_t^W for N periods ($N = 40$)

[1] We thank Martin Andreasen for sharing the codes for the pruning of DSGE models approximated to the 3^{rd}-order. See Andreasen et al. (2013) for details.

2. Perform simulation Y_t^1 starting from initial conditions Y_0 and using ε_t^W
3. Add one standard deviation to ε_t^W in period 1 and get $\tilde{\varepsilon}_t^W$
4. Perform simulation Y_t^2 starting from initial conditions Y_0 and using $\tilde{\varepsilon}_t^W$
5. IRF is equal to $Y_t^2 - Y_t^1$
6. Perform $R = 50$ replications of steps 1) to 5) and report the average IRF

D Micro uncertainty - A Comparative Steady State Exercise

This appendix presents a comparative statics exercise to get a deeper insight of the mechanism through which micro uncertainty affects the real economy. Specifically, we analyze the effect of changes in the steady-state value of the standard deviation of entrepreneurial idiosyncratic productivity ($\bar{S}$) on the steady-state value of other variables in our model economy. Such a simple static exercise is useful for a better understanding of the impulse responses reported in the main text of the paper.

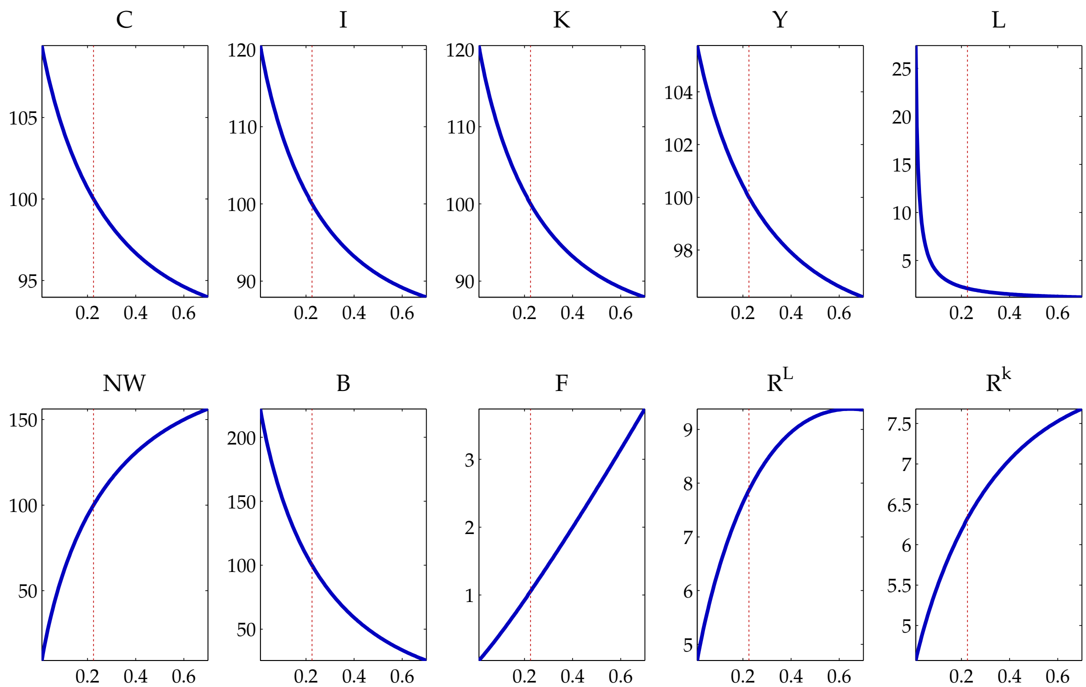
We consider a wide range of steady-state standard deviations of idiosyncratic productivity, namely $\bar{S} = [0.01, 0.70]$. Then, we solve the microeconomic problem together with the steady state of our model with the algorithm described in the Appendix. In this way we pin down the steady-state leverage ratio (L), the threshold value for the idiosyncratic shock ($\bar{\omega}$), and entrepreneurial real income from owning one unit of capital (y^k). Once the steady-state level of these variables are determined, we can solve for the steady-state value of all other variables in our model, given our baseline calibration.

Figure 1 displays how the steady-state level of some key variables in our model varies to changes in the steady-state value of S. Note that all variables expressed in levels are re-scaled to be equal to 100 for our baseline calibration ($\bar{S} = 0.225$); interest rates are in annualized percent, while the leverage ratio is not rescaled.

As described in the main text, an increase in $\bar{S}$ is associated with an increase in the frequency of entrepreneurial default (F) which also increases banks' expected costs associated with bankruptcies. As a result, banks charge a higher spread on the risk free rate and lending rates (R^L) increase, the aggregate level of borrowing in the economy (B) falls and so do entrepreneurs' purchases of unfinished capital (K): with a lower level of capital in the economy its rental rate of return is higher (R^k).

Note that, intuitively, entrepreneurs should try to leverage up to benefit from the higher rental rate of capital. However, as already noted above, as $\bar{S}$ rises entrepreneurs face increasing interest rates (R^L), which would induce entrepreneurs to reduce borrowing and, consequently, also leverage. In a on line appendix to their paper, Christiano et al. (2014) put forth this same issue and analyze it with a similar exercise. They first characterize the equilibrium in the loans market analytically in the *Risk spread - Leverage* space. Then, holding the aggregate return on capital (R^k) fixed, they show that entrepreneurs facing an exogenous increase in $\bar{S}$ would optimally choose a loan contract with a higher interest rate and lower leverage. However, they

Figure 1 Micro Uncertainty: A Comparative Steady State Exercise



Note. The charts display the effect of micro uncertainty on the steady state level of the model economy. On the horizontal axis is the steady state value of entrepreneurial idiosyncratic productivity ($\bar{S}$). On the vertical axis is the steady state value of consumption (C), investment (I), capital (K), total output (Y), leverage (L), net worth (NW), total borrowing (B), default probability (F), lending rates (R^L) and rental rate of capital (R^k).

also suggest that the result of this partial equilibrium exercise could be muted by the increase in the aggregate return on capital, that would instead push entrepreneurs to increase their leverage. In an additional partial equilibrium exercise they show that this is indeed the case: holding $\bar{S}$ fixed, an exogenous increase of R^k relative to the risk free interest rate leads to an increase in leverage, therefore muting the negative impact on leverage of a jump in $\bar{S}$. In their numerical experiments, however, they find that the first effects always dominates.

In our exercise we let the rental rate of capital to be determined jointly with all other variables in our model and, consistently with Christiano et al. (2014)'s conjecture, we find that the effect of lending rates predominates on the effect of rental rate of capital. In fact, Figure 1 shows that in the face of increasing $\bar{S}$ —and therefore of increasing interest rates but also of increasing aggregate returns on capital— entrepreneurs optimally choose loans contracts with lower leverage (L).

Note that when $\bar{S}$ approaches zero, leverage is very sensitive to changes in $\bar{S}$. Intuitively, when the variance of the idiosyncratic shock approaches zero entrepreneurs try to leverage up to infinity since their profits are unbounded and the credit friction is not binding. Analytically, this can be easily understood by recalling that leverage is defined as the ratio between capital and net worth and by observing that entrepreneurs optimally reduce their net worth (NW) as

$\bar{S}$ approaches zero.

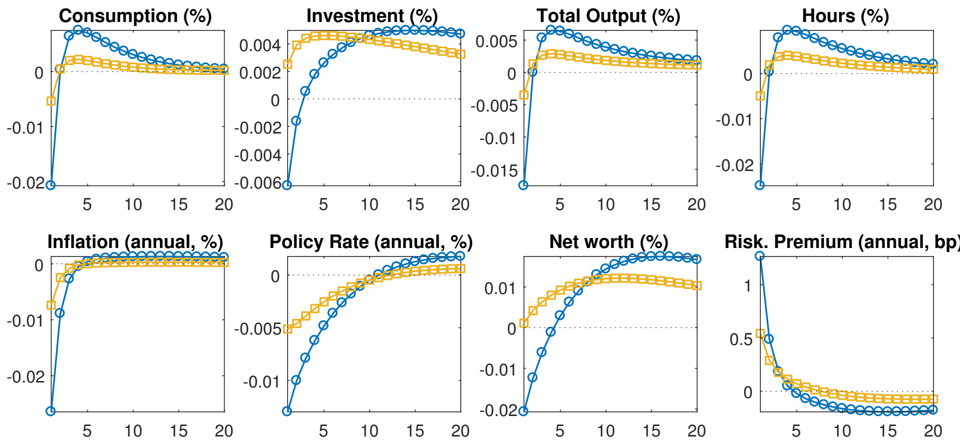
Finally, and not surprisingly, all relevant macroeconomic aggregates are decreasing in $\bar{S}$. Specifically, consumption, investment, and total output are lower for larger values of the standard deviation of idiosyncratic productivity.

E Additional Results

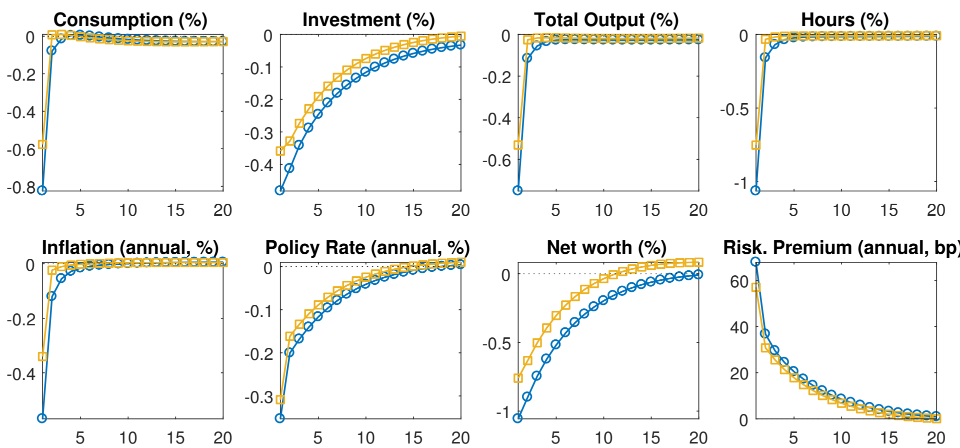
In the main text of the paper we investigate the role of households' preferences, risk aversion and monetary policy response for the transmission of uncertainty shocks. In this Section we report some additional results that complement the ones reported in the paper.

Figure 2 The Impact of Uncertainty Shocks: the Role of Monetary Policy

Panel A: Macro Uncertainty



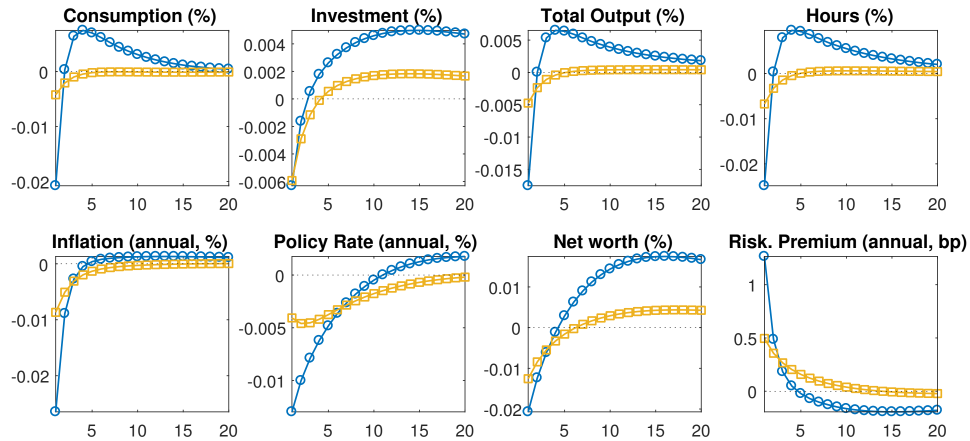
Panel B: Micro Uncertainty



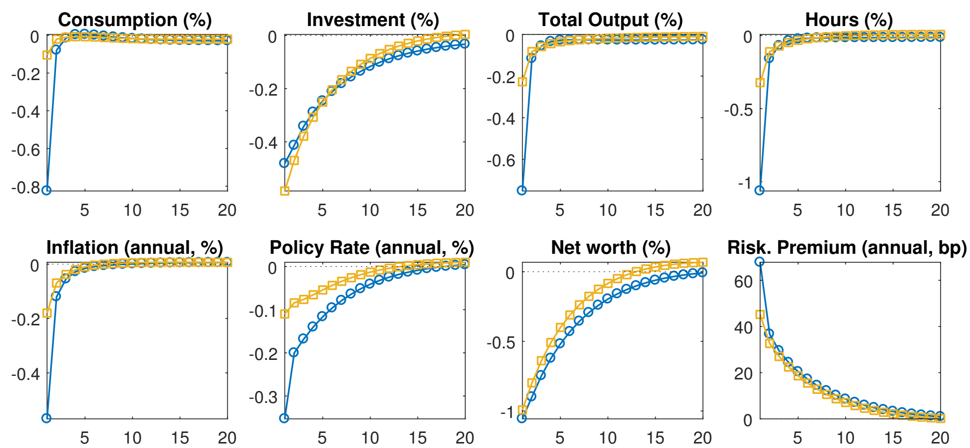
Note. Impulse response functions (IRFs) to a 1 standard deviation increase in macro uncertainty (panel A) and in micro uncertainty (panel B). The IRFs are computed with respect to the ergodic mean of the variables of interest. All responses are in percent, except for the risk premium which is in basis points. Differently from the baseline, the coefficient on inflation in the Taylor rule has been set to $\phi^{\pi} = 3$.

Figure 3 The Impact of Uncertainty Shocks: the Role of Households Preferences

Panel A: Macro Uncertainty



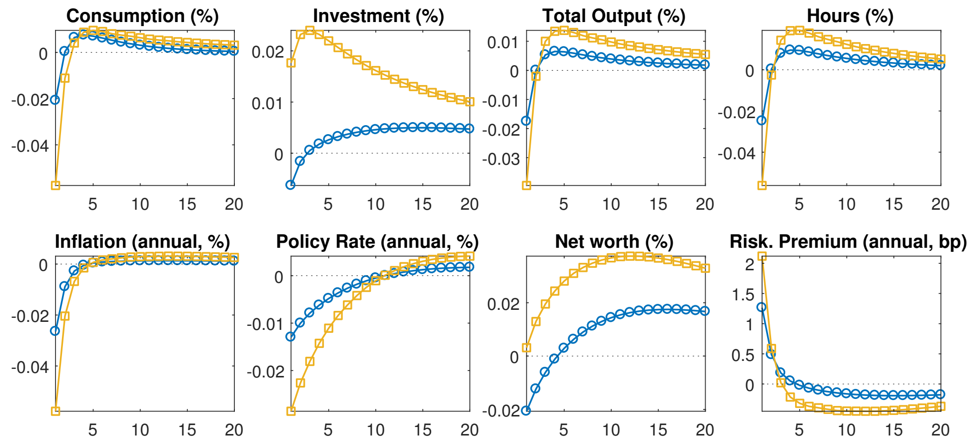
Panel B: Micro Uncertainty



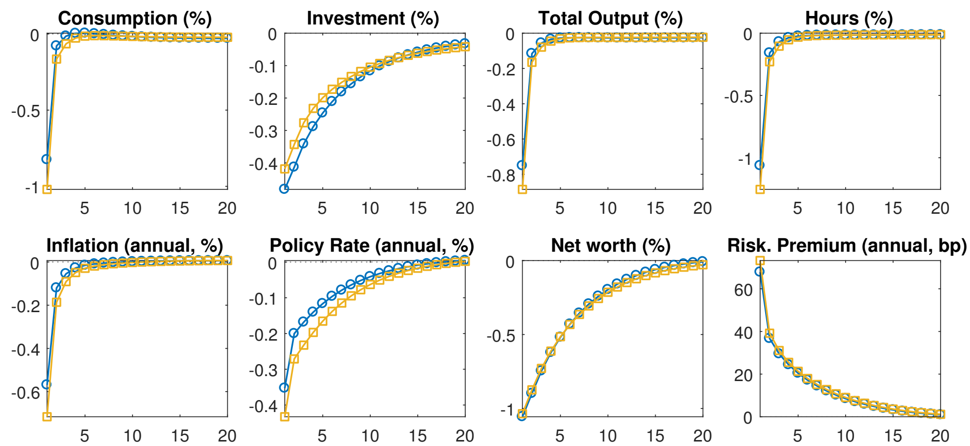
Note. Impulse response functions (IRFs) to a 1 standard deviation increase in macro uncertainty (panel A) and in micro uncertainty (panel B). The IRFs are computed with respect to the ergodic mean of the variables of interest. All responses are in percent, except for the risk premium which is in basis points. Differently from the baseline, we obtain these responses with log-separable preferences between consumption and leisure as in King et al. (1988).

Figure 4 The Impact of Uncertainty Shocks: the Role of Risk Aversion

Panel A: Macro Uncertainty



Panel B: Micro Uncertainty



Note. Impulse response functions (IRFs) to a 1 standard deviation increase in macro uncertainty (panel A) and in micro uncertainty (panel B). The IRFs are computed with respect to the ergodic mean of the variables of interest. All responses are in percent, except for the risk premium which is in basis points. Differently from the baseline, the coefficient of relative risk aversion has been set to $\varrho = 5$.

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