

Uncertainty and Economic Activity: A Multi-Country Perspective

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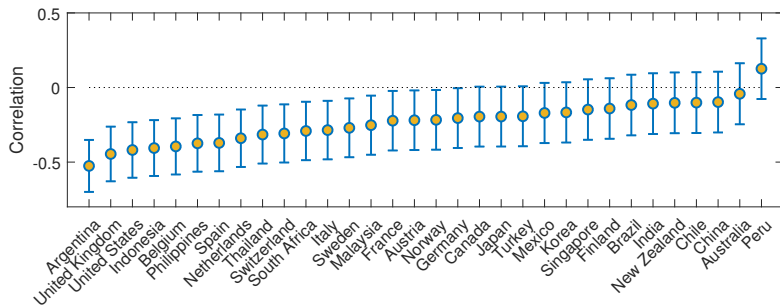
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*The views expressed in this paper do not necessarily reflect the position of the Bank of England.

Strong and robust association between measures of uncertainty and economic activity



- ▶ Data for 32 countries, covering about 90 percent of world GDP
 - * **Uncertainty proxy:** Quarterly realized equity market volatility computed from daily returns.
 - * **Economic activity proxy:** Quarterly real GDP growth.

But difficult to interpret

- ▶ Theory is ambiguous about the direction of causation.
 - * Uncertainty dampens activity (precautionary savings, irreversible investments, financial frictions,...).
 - * Recessions can also increase uncertainty (financial and information frictions).
- ▶ Question of whether uncertainty *causes* GDP to contract (and by how much) is important for policy makers.

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- ▶ Question of whether uncertainty *causes* GDP to contract (and by how much) is important for policy makers.
- ▶ Identification problem typically tackled in a single-country framework.

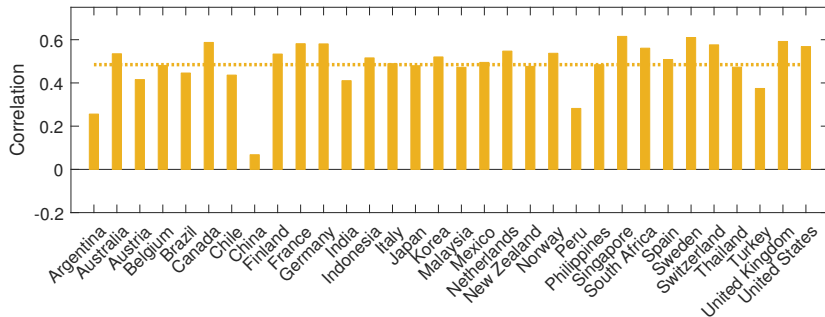
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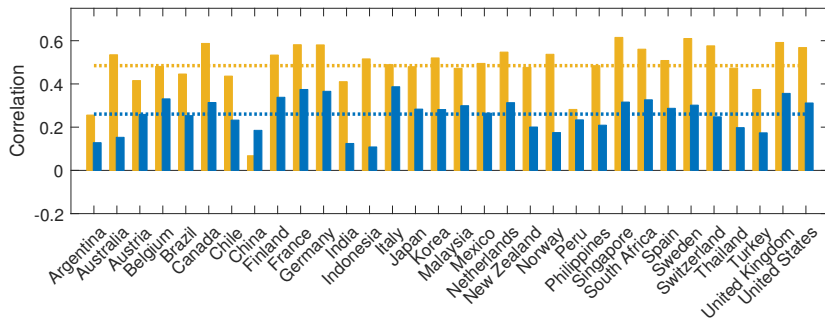
FIGURE: Average pairwise correlation of volatility (yellow bars)



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FIGURE: Average pairwise correlation of volatility (yellow bars) and GDP growth (blue bars).



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- ▶ Proposes a novel common factor approach in a multi-country setting to analyze the relation between uncertainty and economic activity.
- ▶ Empirically, there are two identification problems:
 - [1] Identification of the common factors.
 - [2] Identification of the country-specific shocks.

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- ▶ Proposes a novel common factor approach in a multi-country setting to analyze the relation between uncertainty and economic activity.
- ▶ Empirically, there are two identification problems:
 - [1] Identification of the common factors.
 - [2] Identification of the country-specific shocks.
- ▶ This paper contributes to [1]
 - * Identification of a **growth** and a **financial** factors exploiting different patterns of cross-country correlations of volatility and GDP growth rates.

This paper: Main Findings

- [1] The growth factor accounts for the bulk of the (unconditional) negative correlation between volatility and GDP growth for most countries.
 - * Part of the explanatory power attributed to volatility shocks in single-country studies is due to omitted common factors.

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 - * Part of the explanatory power attributed to volatility shocks in single-country studies is due to omitted common factors.
- [2] Forecast error variance of volatility is largely explained by shocks to the financial factor and shocks to country-specific volatility itself.
- [3] Shocks to the financial factor explain a non-trivial ($\sim 10\%$) share of GDP growth forecast error variance, and can have strong and persistent contractionary effects.

Related literature

- ▶ **Uncertainty can respond to the business cycle** [Ludvigson, Ma, Ng (2017), Berger, Dew-Becker and Giglio (2017), Berger and Vavra (2017)]
 - * We impose restrictions on the cross-section rather than on individual countries.
 - * Restrictions consistent with observable properties of the data.
 - * We obtain similar results with very different approach.

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 - * Multi-country framework, as opposed to a set of countries considered in isolation.

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 - * Multi-country framework, as opposed to a set of countries considered in isolation.
- ▶ **Stylized facts on cross-country correlations in intl. finance literature** [Tesar (1995), Colacito and Croce (2011), Lewis and Liu (2015)]
 - * Focus on “equity volatility-growth” relationship, rather than “equity returns-consumption” one.

Outline

[1] **Theoretical model**

Equity returns and volatility in a multi-country business cycle model.

[2] **Empirical framework**

A multi-country factor model for volatility and the business cycle.

[3] **Data & Empirical results**

[4] **Conclusions**

Theoretical model: Multi-country Lucas tree model

- ▶ Multi-country version of the [Lucas \(1978\)](#) tree model where country-specific output growth is driven by two processes:
 - * Persistent global growth factor with stochastic volatility and heterogeneous loadings.
 - * Country-specific business cycle component with stochastic volatility.

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 - * Persistent global growth factor with stochastic volatility and heterogeneous loadings.
 - * Country-specific business cycle component with stochastic volatility.
- ▶ Two steps:
 - [1] Solve for the risk free rate as a function of global growth factor and its volatility.
 - [2] Solve for the country-specific equity returns using [Campbell and Shiller \(1989\)](#) approximation and [Bansal and Yaron \(2004\)](#) approach.

Model details

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 - * Persistent global growth factor with stochastic volatility and heterogeneous loadings.
 - * Country-specific business cycle component with stochastic volatility.
- ▶ Assumptions:
 - * Country-specific business cycle components are weakly correlated across countries.
 - * No country is dominant in the world economy (granularity).

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 - * Persistent global growth factor with stochastic volatility and heterogeneous loadings.
 - * Country-specific business cycle component with stochastic volatility.
- ▶ Main insights:
 - * Global growth factor is only driver of cross-country correlation of output growth.
 - * At least one additional common shock (a composite of second and higher-order moment shocks) is required to explain the cross-country correlation of equity volatility.

Empirical Framework

A multi-country econometric framework

- ▶ First-order panel vector autoregressive (PVAR) model in Δy_{it} and v_{it} for $i = 1, 2, \dots, N$:

$$\begin{aligned}\Delta y_{it} &= a_{iy} + \phi_{i,11}v_{i,t-1} + \phi_{i,12}\Delta y_{i,t-1} + e_{iy,t}, \\ v_{it} &= a_{iv} + \phi_{i,21}v_{i,t-1} + \phi_{i,22}\Delta y_{i,t-1} + e_{iv,t}.\end{aligned}$$

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- ▶ Consistent with the theoretical model and the stylized facts, we posit the following unobservable common-factor representation for $e_{iy,t}$ and $e_{iv,t}$:

$$\begin{aligned}e_{iy,t} &= \gamma_i\zeta_t + \varepsilon_{it}, \\ e_{iv,t} &= \lambda_i\zeta_t + \theta_i\xi_t + \eta_{it}.\end{aligned}$$

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- ▶ **Objective** Identification of factors (ζ_t, ξ_t) and loadings $(\lambda_i, \gamma_i, \theta_i)$.

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- ▶ **Our approach** Identification is achieved by placing restrictions on the *cross-country* correlations of e_{iy} and e_{iv} , namely $Corr(e_{iy}, e_{jy})$ and $Corr(e_{iv}, e_{jv}) \forall i, j$.

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- ▶ **Note #1** *Within-country* correlation $\text{Corr}(\varepsilon_{it}, \eta_{it})$ for $i = 1, 2, \dots, N$ is left unrestricted. To identify the country-specific shocks we will use auxiliary assumptions typically used in the literature.

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- ▶ **Note #2** In a single-country model, the common factors (ζ_t, ξ_t) or their loadings ($\lambda_i, \gamma_i, \theta_i$) cannot be identified even if it is assumed that $\text{Corr}(\varepsilon_{it}, \eta_{it}) = 0$.

Definitions & Identifying assumptions

Definitions

- ▶ Define global GDP growth ($\Delta\bar{y}_{\omega,t}$) and global volatility ($\bar{v}_{\omega,t}$) as weighted (w_i) averages over a large number of countries:

$$\Delta\bar{y}_{\omega,t} = \sum_{i=1}^N w_i \Delta y_{it}, \quad \bar{v}_{\omega,t} = \sum_{i=1}^N w_i v_{it}$$

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Identifying assumptions

1. Common shocks & Loadings: pervasive factors (ζ_t for both volatility and activity, ξ_t for volatility only).
2. Weights: granularity (weights w_i are not dominated by a few cross-section units).
3. Cross-sectional correlations: weak dependence of country-specific innovations (pairwise correlations of ε_{it} and η_{it} tend to zero).

The common growth shock (ζ_t): Identification by aggregation

- ▶ For simplicity, and to focus on our novel approach, consider first a 'static' case:

$$\begin{aligned}\Delta y_{it} &= \gamma_i \zeta_t + \varepsilon_{it}, \\ v_{it} &= \lambda_i \zeta_t + \theta_i \xi_t + \eta_{it}.\end{aligned}$$

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- ▶ In this simple case, the factor ζ_t can be identified by $\bar{y}_{\omega,t}$ (up to a constant):

$$\zeta_t = \gamma^{-1} \Delta \bar{y}_{\omega,t} + O_p\left(N^{-1/2}\right).$$

- ▶ **Intuition** The GDP growth innovations ε_{it} only include idiosyncratic risk that wash out when N is large.

The common growth shock (ζ_t): Identification by aggregation

- **Proof** Consider the weighted average of the country-specific systems:

$$\begin{aligned}\Delta \bar{y}_{\omega,t} &= \gamma \zeta_t + \bar{\varepsilon}_{\omega,t}, \\ \bar{v}_{\omega,t} &= \lambda \zeta_t + \theta \xi_t + \bar{\eta}_{\omega,t}.\end{aligned}$$

where $\bar{\eta}_{\omega,t} = \mathbf{w}'\boldsymbol{\eta}_t$ and $\bar{\varepsilon}_{\omega,t} = \mathbf{w}'\boldsymbol{\varepsilon}_t$. Under the above assumptions it now readily follows that:

$$\zeta_t = \frac{\Delta \bar{y}_{\omega,t}}{\gamma} + \underbrace{\frac{\bar{\varepsilon}_{\omega,t}}{\gamma}}_{O_p(N^{-1/2})}$$

and for N sufficiently large ζ_t can be approximated by $\Delta \bar{y}_{\omega,t}$ (up to a constant), as

$$Var(\bar{\varepsilon}_{\omega,t}) = \mathbf{w}'\boldsymbol{\Sigma}_\varepsilon\mathbf{w} \leq (\mathbf{w}'\mathbf{w}) \varrho_{\max}(\boldsymbol{\Sigma}_\varepsilon)$$

and so $Var(\bar{\varepsilon}_{\omega,t}) = O(\mathbf{w}'\mathbf{w}) = O(N^{-1})$.

The common growth shock (ζ_t): Remarks

Remark Interpretation of ζ_t

Because ζ_t is the same as world growth rescaled, we label it a common “growth” shock, in line with our theoretical derivations.

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Remark The role of N

The triangular factor structure alone does not identify ζ_t . To get identification we need large N , otherwise we would not be able to disentangle ζ_t from $\bar{\varepsilon}_{\omega,t}$.

The common financial shock (ξ_t)

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- ▶ But doing so permits exploring other properties of the data.
- ▶ Conditional on ζ_t , for N large enough, ξ_t is determined by:

$$\xi_t = \frac{\bar{v}_{\omega,t}}{\theta} - \frac{\lambda}{\theta\gamma} \Delta \bar{y}_{\omega,t} + \underbrace{\frac{\bar{\eta}_{\omega,t}}{\theta}}_{O_p(N^{-1/2})}$$

- ▶ **Proof** Same logic as above applied to η_{it} . Factor ξ_t can be identified from the data as a linear combination of $\Delta \bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$ (up to an orthonormal transformation).

The common financial shock (ξ_t): Remarks

Remark We label ξ_t a global ‘financial’ shock to highlight its role in capturing all higher-order terms present in our theoretical model once we account for the common growth factor shock, ζ_t .

ξ_t could also reflect changes in non-fundamental aspects of financial markets, such as over-reactions to news due to excessive optimism/pessimism or bubble components, ruled out by the theoretical model.

A dynamic multi-country heterogeneous model

- ▶ Consider the following first-order dynamic version of the above static model:

$$\begin{aligned}\Delta y_{it} &= a_{iy} + \phi_{i,11}v_{i,t-1} + \phi_{i,12}\Delta y_{i,t-1} + \gamma_i\zeta_t + \varepsilon_{it} \\ v_{it} &= a_{iv} + \phi_{i,21}v_{i,t-1} + \phi_{i,22}\Delta y_{i,t-1} + \lambda_i\zeta_t + \theta_i\xi_t + \eta_{it}\end{aligned}$$

- ▶ Identification of the common factors in this dynamic and heterogeneous setting is sensibly more challenging.
- ▶ Stronger assumptions on PVAR coefficients ($\phi_{i,rs}$) and higher order lags of $(\bar{v}_{\omega,t}, \Delta\bar{y}_{\omega,t})$ are needed.

Skip derivations

Dynamic model: Assumptions

- ▶ **Factor loadings** The factor loadings λ_i , θ_i , and γ_i (i.e., the non-zero elements of $\mathbf{\Gamma}_i$) are independently distributed across i , and of the common factors, \mathbf{f}_t , for all i and t , with non-zero means λ , θ , and γ , and second-order moments. Furthermore:

$$\mathbf{\Gamma} = \mathbb{E}(\mathbf{\Gamma}_i) = \begin{pmatrix} \gamma & 0 \\ \lambda & \theta \end{pmatrix}$$

- ▶ **Coefficients** The constants \mathbf{a}_i are bounded, Φ_i and $\mathbf{\Gamma}_i$ are independently distributed for all i , the support of $\varrho(\Phi_i)$ lies strictly inside the unit circle, for $i = 1, 2, \dots, N$, and the inverse of the polynomial $\mathbf{\Lambda}(L) = \sum_{\ell=0}^{\infty} \mathbf{\Lambda}_\ell L^\ell$, where $\mathbf{\Lambda}_\ell = \mathbb{E}(\Phi_i^\ell)$ exists and has exponentially decaying coefficients, namely $\|\mathbf{\Lambda}_\ell\| \leq K\rho^\ell$, where K is a fixed constant and $0 < \rho < 1$.

Back

Dynamic model: Identification of the factors

- **Proposition** Common factors ζ_t and ξ_t are given by:

$$\zeta_t = b_\zeta + \gamma^{-1} \Delta \bar{y}_{\omega,t} + \sum_{\ell=1}^{\infty} \mathbf{c}'_{1,\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + O_p \left(N^{-1/2} \right),$$

$$\xi_t = b_\xi + \theta^{-1} \left(\bar{v}_{\omega,t} - \frac{\lambda}{\gamma} \Delta \bar{y}_{\omega,t} \right) + \sum_{\ell=1}^{\infty} \mathbf{c}'_{2,\ell} \bar{\mathbf{z}}_{\omega,t-\ell} + O_p \left(N^{-1/2} \right),$$

where $\mathbf{z}_{it} = (\Delta y_{it}, v_{it})'$.

Proof

Dynamic model: Consistent estimation of orthogonal factors

- **Proposition** Consistent estimators of the common shocks, denoted by $\hat{\zeta}$ and $\hat{\xi}$, can be obtained as residuals from the following OLS regressions:

$$\begin{aligned}\hat{\zeta} &= \Delta \bar{y}_\omega - \bar{Z}_\omega \hat{C}_1, \\ \hat{\xi} &= \bar{v}_\omega - \hat{\lambda} \hat{\zeta} - \bar{Z}_\omega \hat{C}_2,\end{aligned}$$

where $\zeta = (\zeta_1, \zeta_2, \dots, \zeta_T)'$, $\xi = (\xi_1, \xi_2, \dots, \xi_T)'$,
 $\bar{Z}_\omega = (\tau_T, \bar{z}_{\omega,-1}, \bar{z}_{\omega,-2}, \dots, \bar{z}_{\omega,-p})$, $\bar{z}_{\omega,-l} = (\Delta \bar{y}_{\omega,-l} \bar{v}_{\omega,-l})$,
 $\Delta \bar{y}_{\omega,-l} = (\Delta \bar{y}_{\omega,1-l}, \Delta \bar{y}_{\omega,2-l}, \dots, \Delta \bar{y}_{\omega,T-l})'$, $\Delta \bar{y}_\omega = \Delta \bar{y}_{\omega,0}$,
 $\bar{v}_{\omega,-l} = (\bar{v}_{\omega,1-l}, \bar{v}_{\omega,2-l}, \dots, \bar{v}_{\omega,T-l})'$, $\bar{v}_\omega = \bar{v}_{\omega,0}$, and p denotes a suitable number of lags (or truncation order).

Proof

Dynamic model: Factor-augmented large VAR

- ▶ Theoretical results carry through a fully heterogeneous dynamic version of the model.
- ▶ Recalling that $\bar{z}_{\omega,t} = (\bar{v}_{\omega,t}, \Delta \bar{y}_{\omega,t})$, the country-specific models with orthonormal factors can be written as:

$$\begin{aligned}\Delta y_{it} &= \phi_{i,11} v_{i,t-1} + \phi_{i,12} \Delta y_{i,t-1} + \beta_{i,11} \hat{\zeta}_t + \sum_{l=1}^p \psi'_{\Delta y,il} \bar{z}_{\omega,t-l} + \varepsilon_{it} \\ v_{it} &= \phi_{i,21} v_{i,t-1} + \phi_{i,22} \Delta y_{i,t-1} + \beta_{i,21} \hat{\zeta}_t + \beta_{i,22} \hat{\xi}_t + \sum_{l=1}^p \psi'_{v,il} \bar{z}_{\omega,t-l} + \eta_{it}\end{aligned}$$

- ▶ Country-specific models can then be combined in a large model of the global economy.

Data & Empirical Results

Data

- ▶ Balanced panel of GDP growth and equity volatility for 32 countries from 1993:Q1 to 2016:Q4.

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- ▶ We compute the realized volatility for country i in quarter t as:

$$v_{it} = \log \left(\sqrt{\sum_{\tau=1}^{D_t} (r_{it}(\tau) - \bar{r}_{it})^2} \right)$$

where $r_{it}(\tau) = \Delta \ln P_{it}(\tau)$, and $\bar{r}_{it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{it}(\tau)$ is the average daily price changes in the quarter t , and D_t is the number of trading days in quarter t .

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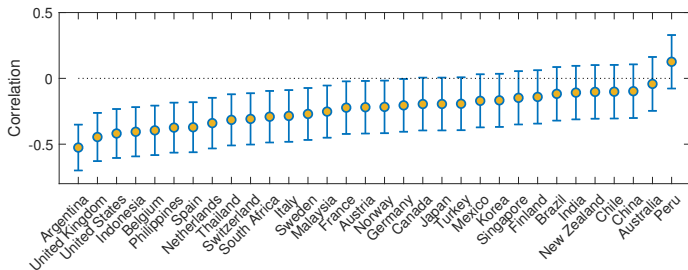
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- ▶ In recent literature focus has shifted to implied volatility measures from option prices (e.g., VIX). But:
 - * Not available for a large number of countries over a long period of time.
 - * [Berger, Dew-Becker, Giglio \(2017\)](#): conditional on realized volatility, VIX is not associated with indicators of economic activity.

Empirical results

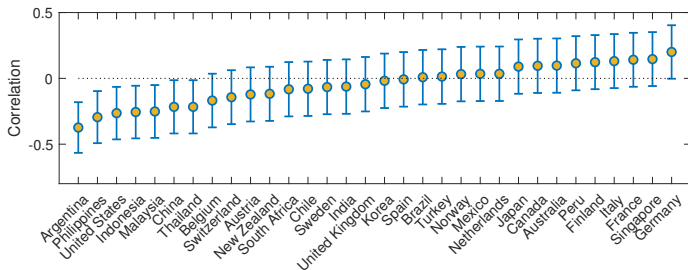
- [R1] Within-country *conditional* correlation of volatility and GDP growth.
- [R2] Omitted variable bias.
- [R3] IRFs and FEVDs to factors and country-specific shocks.
- [R4] Interpretation of the factors.
- [R5] Evidence in support of identifying assumptions (cross-country correlations).

[R1] Within-country correlation of volatility and growth: unconditional



- Unconditional correlation between Δy_{it} and v_{it} (same as Figure shown in introduction).

[R1] Within-country correlation of volatility and growth: conditioning on common growth shock only

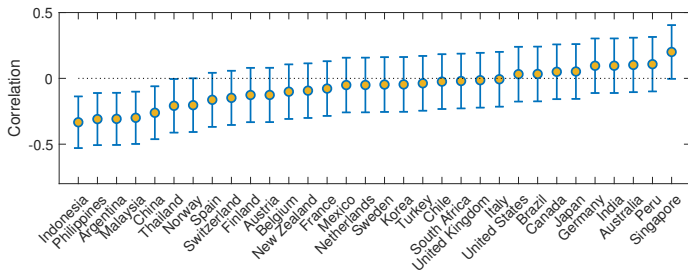


- ▶ Correlation between ε_{it} and u_{it} conditional on growth common shock ($\hat{\zeta}_t$) only.

$$\Delta y_{it} = \beta_{i,21} \hat{\zeta}_t + \text{cross-section averages and lagged values} + \varepsilon_{it}$$

$$v_{it} = \beta_{i,11} \hat{\zeta}_t + \text{cross-section averages and lagged values} + u_{it}$$

[R1] Within-country correlation of volatility and growth: conditioning on both common shocks



- Correlation between ε_{it} and η_{it} conditional on growth ($\hat{\zeta}_t$) and financial ($\hat{\xi}_t$) common shocks

$$\Delta y_{it} = \beta_{i,21} \hat{\zeta}_t + \text{cross-section averages and lagged values} + \varepsilon_{it}$$

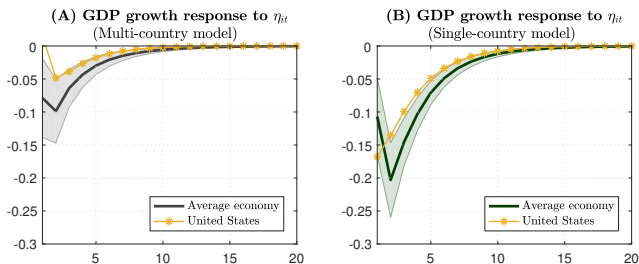
$$v_{it} = \beta_{i,11} \hat{\zeta}_t + \beta_{i,12} \hat{\xi}_t + \text{cross-section averages and lagged values} + \eta_{it}$$

[R2] Omitted variable bias in single-countries VARs

- ▶ Compare IRFs from a single-country VAR (with recursive identification) with those obtained from our multi-country model.

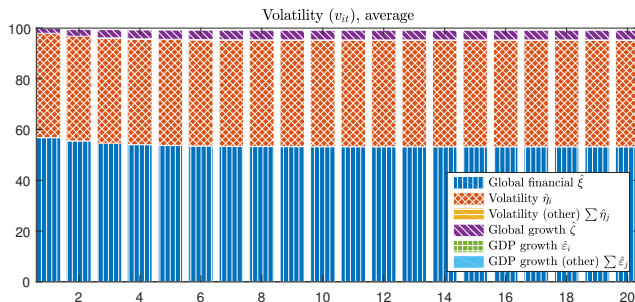
[R2] Omitted variable bias in single-countries VARs

- ▶ Compare IRFs from a single-country VAR (with recursive identification) with those obtained from our multi-country model.
- ▶ Consider a 1 st. deviation shock to country-specific volatility η_{it} .



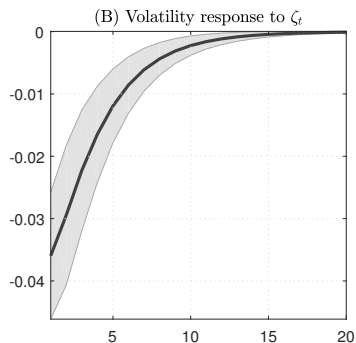
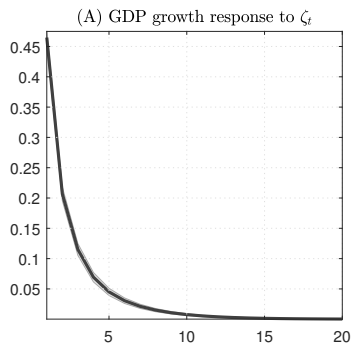
- ▶ Single-country model largely overestimates the impact of domestic shocks on GDP growth.

[R3] Is output growth important for volatility?



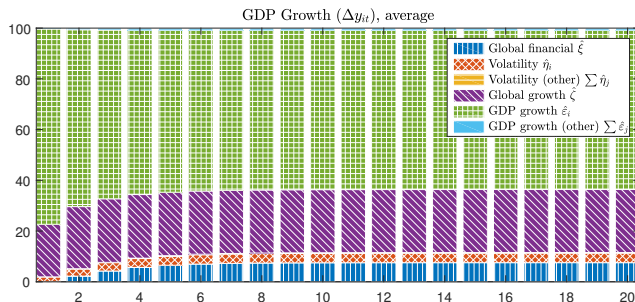
- ▶ Growth factor ($\hat{\zeta}_t$, **purple areas**) and country-specific growth innovations (ε_{it} , **green areas**) jointly explain less than $< 5\%$ of volatility forecast error variance.
- ▶ Volatility is largely “exogenous”.

[R3] Is output growth important for volatility?



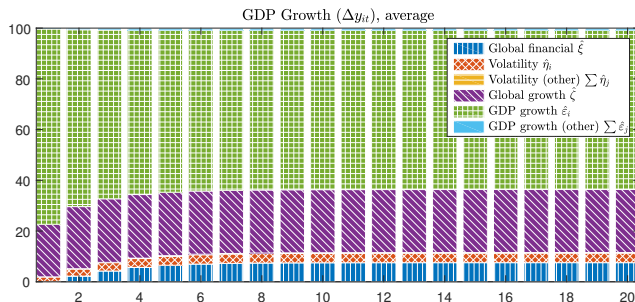
- ▶ Positive growth shock ($\hat{\zeta}_t$) leads to an increase in world GDP growth...
- ▶ ... and to a countercyclical fall in volatility.

[R3] Is volatility harmful for growth?



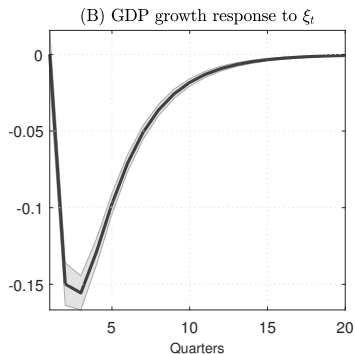
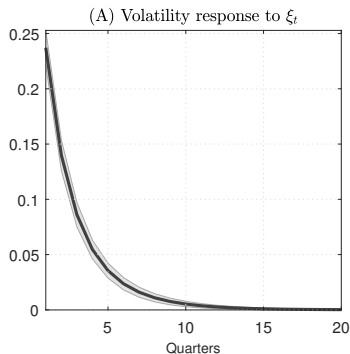
- ▶ Financial factor ($\hat{\xi}_t$, **dark blue areas**) explains a non-negligible share of growth forecast error variance (about $\sim 10\%$).

[R3] Is volatility harmful for growth?



- ▶ Financial factor ($\hat{\xi}_t$, **dark blue areas**) explains a non-negligible share of growth forecast error variance (about $\sim 10\%$).
- ▶ But country-specific volatility shocks (η_{it} , **red areas**) diversified away.

[R3] Is volatility harmful for growth?



- ▶ Adverse financial shock ($\hat{\xi}_t$) leads to an increase in world volatility...
- ▶ ... and to a sharp and persistent contraction of world GDP growth.

[R4] The estimated common shocks: Interpretation

- ▶ Theoretical model provides one way to interpret of $\hat{\zeta}_t$ and $\hat{\xi}_t$, as global growth and financial shocks.
- ▶ Check the consistency of such interpretation with a number of observable proxies.

[R4] The estimated common shocks: Interpretation

Table CORRELATIONS BETWEEN GLOBAL SHOCKS AND OBSERVABLE PROXIES

	Panel A: Global growth shock ($\hat{\zeta}_t$)	
	Global	US
(1) TFP	0.80 ^(a)	0.37
(2) TFP Util. Adj.	0.46 ^(c)	-0.08
(3) Natural rate of interest (r^*)	0.41 ^(a)	0.12
(4) Long-run Risk	0.31	0.27

	Panel B: Global financial shock ($\hat{\xi}_t$)	
	Global	US
(5) MM Uncertainty	0.41 ^(a)	0.06
(6) RS Uncertainty	0.20 ^(c)	0.08
(7) EPU Index	0.46 ^(a)	0.36 ^(a)
(8) LMN Fin. Uncertainty	n.a.	0.32 ^(a)

NOTE. Statistical significance at 1-percent, 5-percent, and 10-percent level is denoted by (a), (b), and (c), respectively. Correlations computed over longest overlapping sample period. Longest possible sample period is 1993:Q1-2016:Q4.

[R4] The estimated common shocks: Interpretation

- ▶ Theoretical model provides one way to interpret of $\hat{\zeta}_t$ and $\hat{\xi}_t$, as *global* growth and financial shocks.
- ▶ Check the consistency of such interpretation with a number of observable proxies.
- ▶ Supportive of an interpretation of the global factors that is:
 - (i) Consistent with the theoretical model.
 - (ii) Not primarily driven by the United States as a dominant economy.

[R5] Are the identifying assumptions on cross-sectional dependence consistent with the data?

- ▶ Estimate country models with growth shock ($\hat{\zeta}_t$) only:

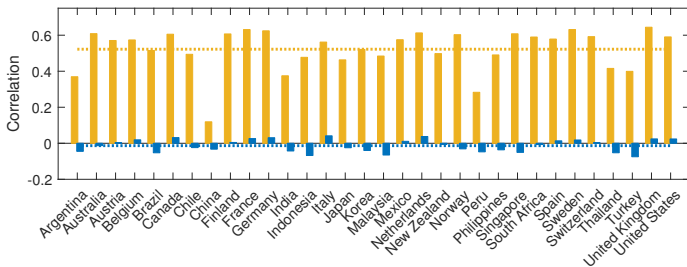
$$\begin{aligned}\Delta y_{it} &= \beta_{i,21} \hat{\zeta}_t + \text{cross-section averages and lagged values} + \varepsilon_{it} \\ v_{it} &= \beta_{i,11} \hat{\zeta}_t + \text{cross-section averages and lagged values} + u_{it}\end{aligned}$$

[R5] Are the identifying assumptions on cross-sectional dependence consistent with the data?

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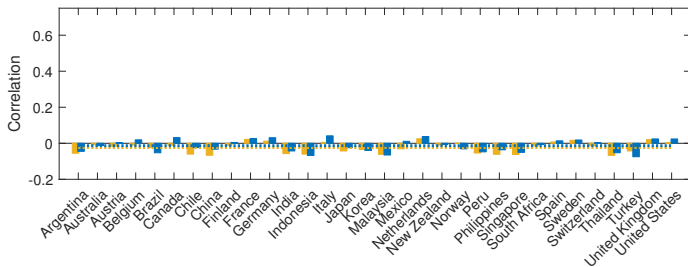
NOTE. Average pairwise correlation of the u_{it} (yellow bars) and the ε_{it} (blue bars).

[R5] Are the identifying assumptions on cross-sectional dependence consistent with the data?

- ▶ Estimate country models with growth ($\hat{\zeta}_t$) and financial ($\hat{\xi}_t$) shocks:

$$\Delta y_{it} = \beta_{i,21} \hat{\zeta}_t + \text{cross-section averages and lagged values} + \varepsilon_{it}$$

$$v_{it} = \beta_{i,11} \hat{\zeta}_t + \beta_{i,12} \hat{\xi}_t + \text{cross-section averages and lagged values} + \eta_{it}$$



NOTE. Average pairwise correlation of the η_{it} (yellow bars) and the ε_{it} (blue bars).

[R5] Tests of cross-sectional dependence don't reject identifying assumptions

- ▶ CD and Exponent of cross-sectional dependence tests [Pesaran (2015), Bailey et al. (2016)].
- ▶ Results in accordance with assumptions of:
 - * Weak/strong cross-sectional dependence of ε_{it}/u_{it} , respectively.
 - * Weak cross-sectional dependence of both ε_{it} and η_{it} .

	<i>CD</i>	Lower 5%	$\hat{\alpha}$	Upper 95%
<i>Data</i>				
v_{it}	104.57	0.94	0.99	1.05
Δy_{it}	55.73	0.87	1.00	1.14
Innovations (conditional on $\hat{\zeta}_t$)				
u_{it}	110.89	0.96	1.00	1.04
ε_{it}	-2.90	0.56	0.62	0.67
Innovations (conditional on $\hat{\zeta}_t$ and $\hat{\xi}_t$)				
η_{it}	-5.12	0.58	0.64	0.70

Robustness

► Granularity assumption:

- * Results are robust when dropping (i) the US, (ii) China, and (iii) both the US and China. [Go](#)
- * Results are not robust when replacing the global factors with US GDP growth and US realized volatility. [Go](#)

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- ▶ Realized versus implied volatility:
 - * Results are robust when using the VIX Index as a measure of volatility for the United States. [Go](#)
- ▶ Alternative assumptions for identification of country-specific shocks:
 - * Results are robust to block-diagonal covariance matrix with recursive (within-country) identification. [Go](#)
 - * Unrestricted covariance matrix. [Go](#)

Conclusions

- ▶ Paper takes a new common-factor approach in a multi-country model to the relation between volatility and growth.
- ▶ Cross-country correlation structure of volatility and growth is exploited to identify a 'growth' factor and a 'financial' factor.
- ▶ Common factors are important to understand relation between volatility and economic activity.

Conclusions

- ▶ Paper takes a new common-factor approach in a multi-country model to the relation between volatility and growth.
- ▶ Cross-country correlation structure of volatility and growth is exploited to identify a 'growth' factor and a 'financial' factor.
- ▶ Common factors are important to understand relation between volatility and economic activity.
- ▶ **Key implication** Part of explanatory power typically attributed to country-specific uncertainty shocks is due to omitted common factors.

Uncertainty and Economic Activity: A Multi-Country Perspective

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¹Bank of England

²University of Southern California and Trinity College, Cambridge

³Johns Hopkins University Carey Business School, CEPR and NBER

4th International Macroeconomics and Finance Conference
April 12-13, 2019

*The views expressed in this paper do not necessarily reflect the position of the Bank of England.

Appendix

A1::Model

A2::Assumptions

Assumptions

- **Assumption 1: Common factors and their loadings** The common unobservable factors, ζ_t and ξ_t , have zero means and unit variances, and are serially uncorrelated. The factor loadings, λ_i , γ_i , and θ_i , are distributed independently across i and from the common factors f_t and g_t for all i and t , with non-zero means λ , γ , and θ ($\lambda \neq 0$, $\gamma \neq 0$, and $\theta \neq 0$), and satisfy the following conditions, for a finite N and as $N \rightarrow \infty$:

$$N^{-1} \sum_{i=1}^N \lambda_i^2 = \mathcal{O}(1) \quad \lambda = \sum_{i=1}^N \dot{w}_i \lambda_i \neq 0$$

$$N^{-1} \sum_{i=1}^N \gamma_i^2 = \mathcal{O}(1) \quad \gamma = \sum_{i=1}^N w_i \gamma_i \neq 0$$

$$N^{-1} \sum_{i=1}^N \theta_i^2 = \mathcal{O}(1) \quad \theta = \sum_{i=1}^N w_i \theta_i \neq 0$$

Back

Assumptions (Cont.)

- **Assumption 2: Aggregation weights** Let $\mathbf{w} = (w_1, w_2, \dots, w_N)'$ and $\hat{\mathbf{w}} = (\hat{w}_1, \hat{w}_2, \dots, \hat{w}_N)'$ be the $N \times 1$ vectors of non-stochastic weights with $w_i, \hat{w}_i > 0$, $\sum_{i=1}^N w_i = 1$ and $\sum_{i=1}^N \hat{w}_i = 1$, such that the following “granularity” conditions are met:

$$\|\mathbf{w}\| = O(N^{-1}), \quad \frac{w_i}{\|\mathbf{w}\|} = O(N^{-1/2})$$

and

$$\|\hat{\mathbf{w}}\| = O(N^{-1}), \quad \frac{\hat{w}_i}{\|\hat{\mathbf{w}}\|} = O(N^{-1/2})$$

for all i .

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Assumptions (Cont.)

- ▶ **Assumption 3: Cross-section correlations** The country-specific innovations, η_{it} and ε_{it} , have zero means and finite variances, and are serially uncorrelated, but can be correlated with each other both within and between countries. Furthermore, denoting the covariance matrices of the $N \times 1$ innovation vectors $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \dots, \varepsilon_{Nt})'$ and $\eta_t = (\eta_{1t}, \eta_{2t}, \dots, \eta_{Nt})'$ by $\Sigma_{\varepsilon\varepsilon} = \text{Var}(\varepsilon_t)$ and $\Sigma_{\eta\eta} = \text{Var}(\eta_t)$, respectively, it is assumed that:

$$\rho_{\max}(\Sigma_{\varepsilon\varepsilon}) = O(1)$$

$$\rho_{\max}(\Sigma_{\eta\eta}) = O(1)$$

Back

A3::Econometric model

Estimating observable and orthogonal factors

- ▶ **Issue** Factors f_t and g_t are unobservable, and even if known, would be correlated with each other
- ▶ For ease of interpretation it is standard to work with the orthogonalized version of the factors
 - * This task is simplified due to the triangular way the factors affect the global variables, $\Delta\bar{y}_{\omega,t}$ and $\bar{v}_{\omega,t}$
- ▶ Proceed recursively
 - * Factor f_t can be identified up to a constant

$$f_t = \frac{\Delta\bar{y}_{\omega,t}}{\gamma} \Rightarrow \hat{\zeta}_t = \Delta\bar{y}_{\omega,t}$$

- * Factor g_t can then be approximated by the residuals of a regression of world volatility $\bar{v}_{\omega,t}$ on world growth

$$g_t = \frac{\bar{v}_{\omega,t}}{\theta} - \frac{\lambda}{\theta\gamma}\Delta\bar{y}_{\omega,t} \Rightarrow \bar{v}_{\omega,t} = \hat{\beta}\Delta\bar{y}_{\omega,t} + \hat{\xi}_t$$

Consistent estimation of orthogonal factors

- ▶ **Proposition 3** Let $\hat{\zeta}_t$ and $\hat{\xi}_t$ be consistent, orthonormalized estimators of f_t and g_t , respectively, where f_t and g_t are defined by (??) and (??). Then, $\hat{\zeta}_t$ can be obtained by re-scaling $\Delta\bar{y}_{\omega,t}$ so that its variance is 1, while $\hat{\xi}_t$ can be obtained as the standardized residual of a least squares regression of $\bar{v}_{\omega,t}$ on $\Delta\bar{y}_{\omega,t}$.
- ▶ **Proof** Consider equation (??) and (??) and set the coefficients $\alpha_g = (\alpha_{1g}, \alpha_{2g})'$, such that $T^{-1} \sum_{t=1}^T \hat{\zeta}_t \hat{\xi}_t = 0$. This yields:

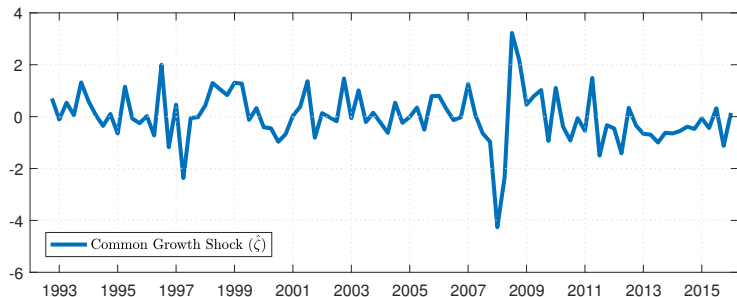
$$\frac{\hat{\alpha}_{2g}}{\hat{\alpha}_{1g}} = \frac{\sum_{t=1}^T \Delta\bar{y}_{\omega,t} \bar{v}_{\omega,t}}{\sum_{t=1}^T \Delta\bar{y}_{\omega,t}^2},$$

which is the OLS estimate of the coefficient on $\Delta\bar{y}_{\omega,t}$ in a regression of \bar{v}_t on $\Delta\bar{y}_{\omega,t}$. Next, set α_f and α_{1g} so that ζ_t and ξ_t have unit in-sample standard deviations.

A3::Additional results

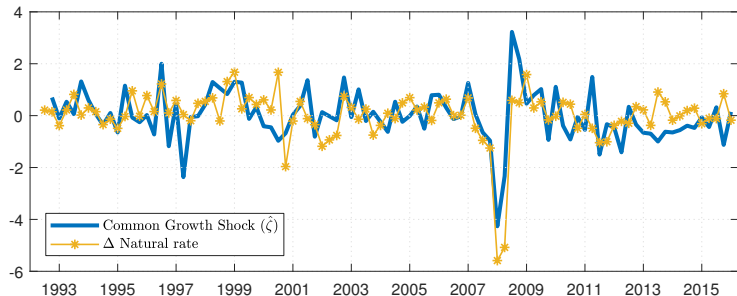
The estimated common growth shock ($\hat{\zeta}$)

- ▶ Common growth shock:



The estimated common growth shock ($\hat{\zeta}$)

- ▶ Common growth shock:

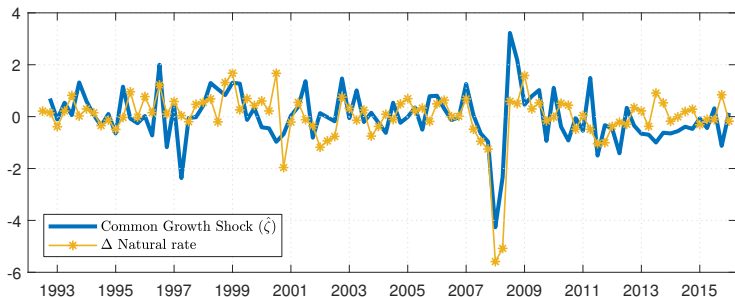


- ▶ Correlated with:

- * Proxy for the world natural rate (0.5) from [Holston, Laubach, Williams \(2017\)](#).

The estimated common growth shock ($\hat{\zeta}$)

► Common growth shock:

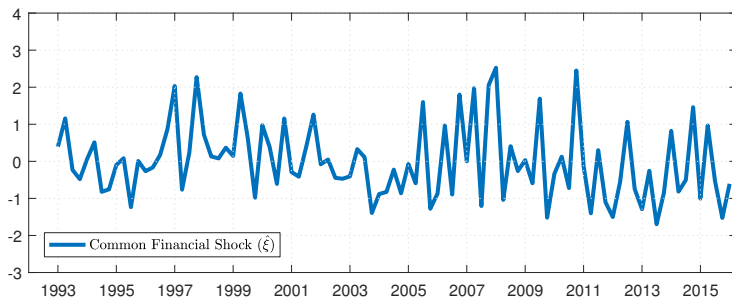


► Correlated with:

- * Proxy for the world natural rate (0.5) from [Holston, Laubach, Williams \(2017\)](#).
- * Proxy for global TFP (0.65) and global utilization adjusted TFP (0.33) from [Huo, Levchenko, Pandalai-Nayar \(2018\)](#).
- * Proxy for global long-run risk (0.29) from [Colacito, Croce, Gavazzoni, Ready \(2018\)](#).

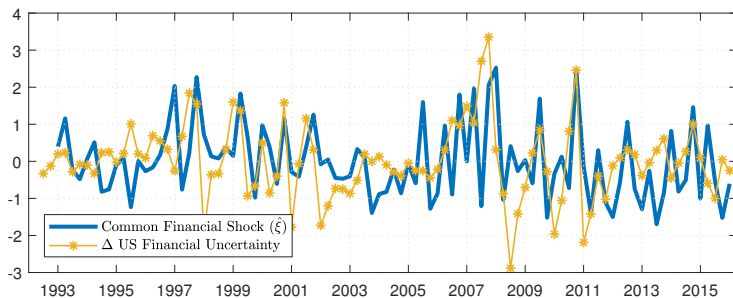
The estimated common financial shock ($\hat{\xi}$)

- ▶ Common financial shock:



The estimated common financial shock ($\hat{\xi}$)

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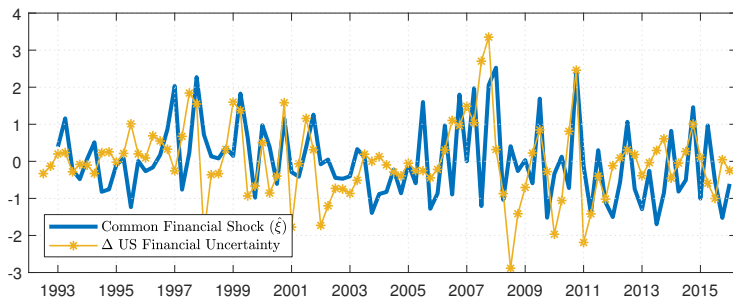


► Correlated with:

- * Financial uncertainty measure (0.43) from [Ludvigson, Ma and Ng \(2017\)](#).

The estimated common financial shock ($\hat{\xi}$)

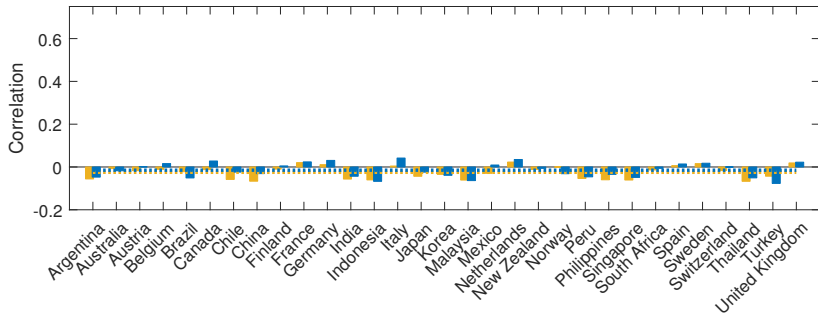
► Common financial shock:



► Correlated with:

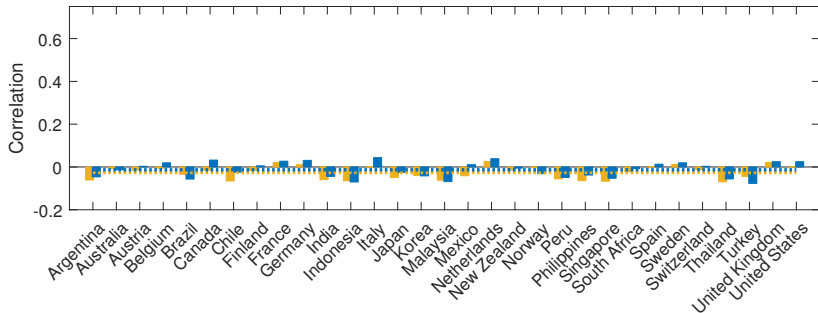
- * Financial uncertainty measure (0.43) from Ludvigson, Ma and Ng (2017).
- * Excess Bond Premium (0.35) Gilchrist and Zakrajsek (2012).

Granularity: No US



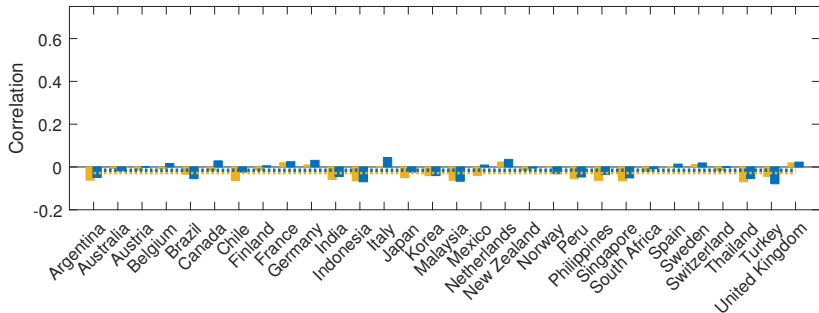
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Granularity: No China



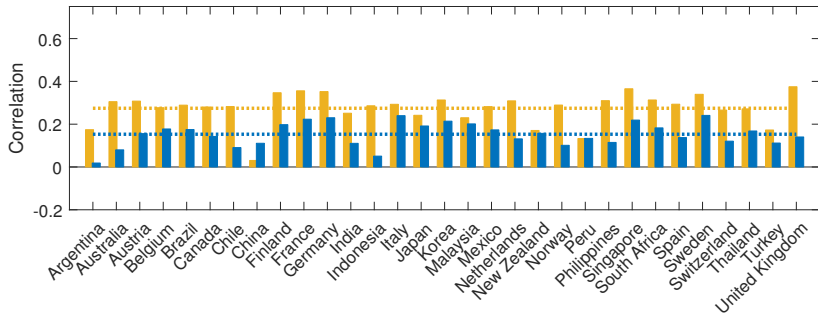
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Granularity: No US & No China



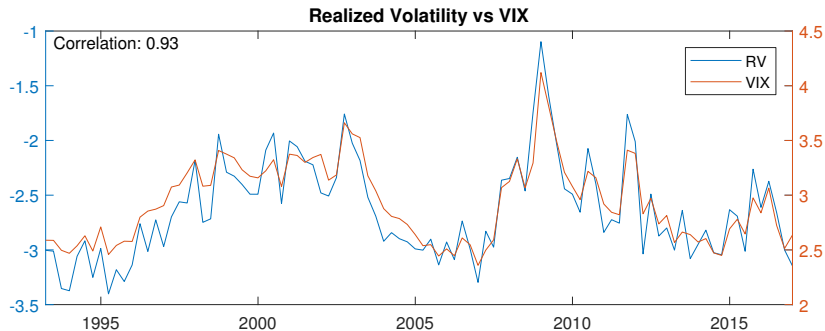
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Granularity: US as the global factor



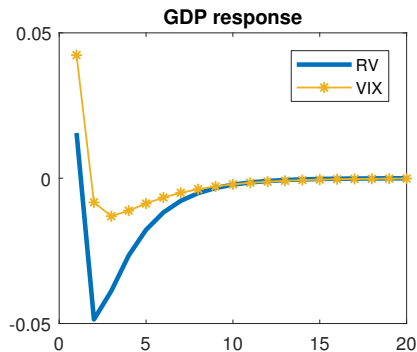
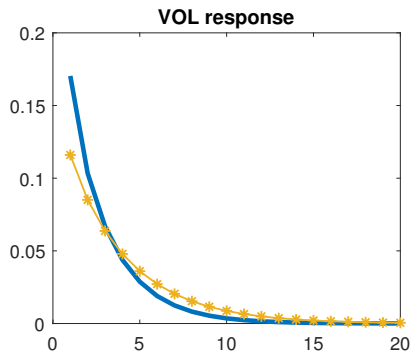
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Comparison between VIX and US realized volatility:Data



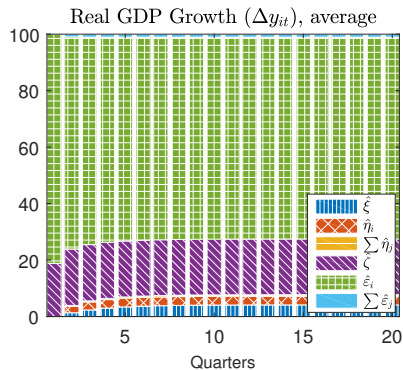
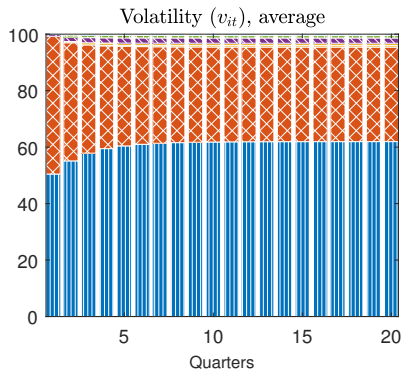
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Comparison between VIX and US realized volatility: IRF



Back

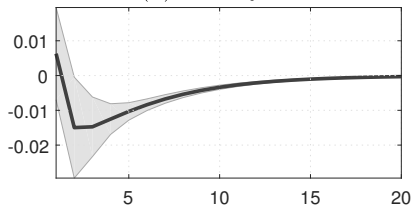
Excluding the global financial crisis: FEVDs



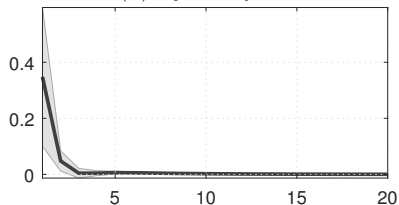
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Excluding the global financial crisis: IRFs

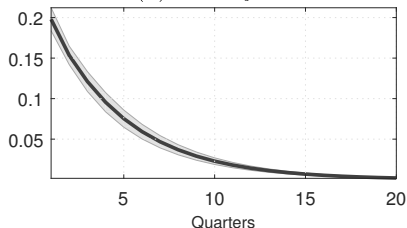
(A) v_{it} to a ζ_t shock



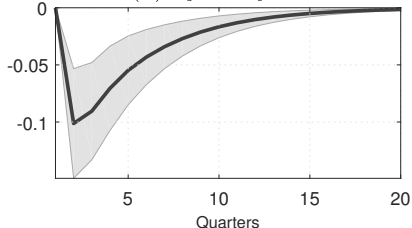
(B) Δy_{it} to a ζ_t shock



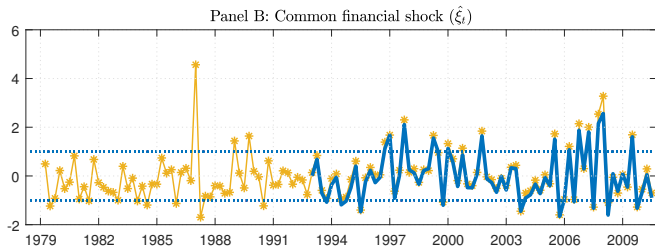
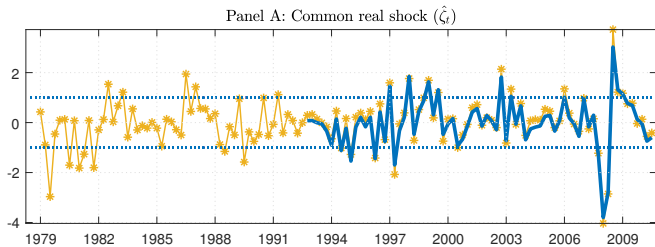
(C) v_{it} to a ξ_t shock



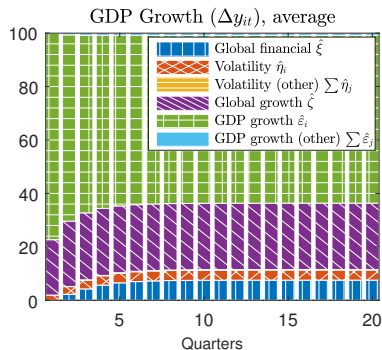
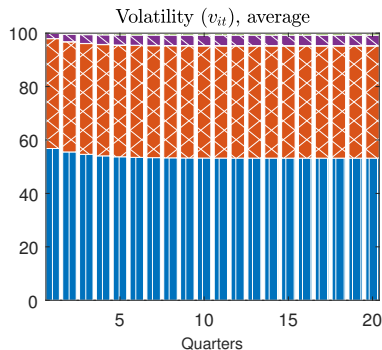
(D) Δy_{it} to a ξ_t shock



Longer sample period: Common Shocks

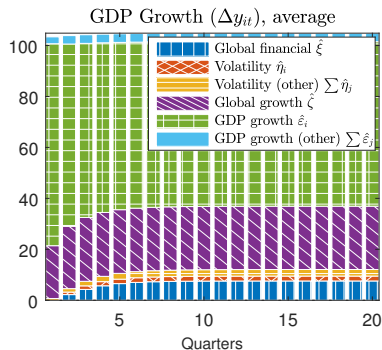
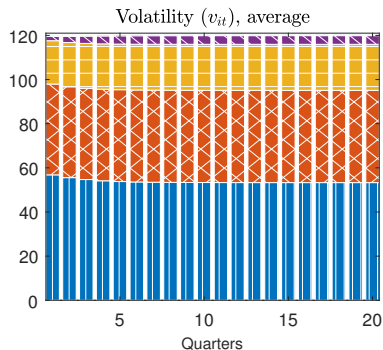


Average FEVD: Block-diagonal covariance matrix



- ▶ Allow for within-country correlation of the innovations (ε_{it} and η_{it}).
- ▶ Orthogonalize with Cholesky decomposition, with volatility ordered first (block-diagonal covariance matrix)

Average GFEVD: Threshold covariance matrix



- ▶ Allow for within- and across-country correlation of the innovations (ε_{it} and η_{it}).
- ▶ Compute Generalized FEVD with threshold covariance matrix.

Country-specific response to the factors

