Global VARs

Ambrogio Cesa-Bianchi

Why do we need a global model?

- Macroeconomic policy analysis requires taking account of the increasing interdependencies that exist across markets and countries
- National economic issues need to be considered from a global perspective
- Policy-makers need to take into account many different channels of transmission

Correlation of GDP growth (1979–2011)

	US	UK	Japan	China	Australia	Canada	Switzerland	Germany
US	1							
UK	0.45	1						
Japan	0.23	0.28	1					
China	0.14	0.20	0.15	1				
Australia	0.43	0.37	0.05	0.12	1			
Canada	0.62	0.34	0.18	0.16	0.35	1		
Switzerland	0.35	0.31	0.31	0.12	0.27	0.36	1	
Germany	0.35	0.34	0.35	0.11	0.04	0.23	0.43	1

Correlation of short-term interest rates (1979–2011)

	US	UK	Japan	China	Australia	Canada	Switzerland	Germany
US	1							
UK	0.88	1						
Japan	0.78	0.88	1					
China	0.42	0.55	0.50	1				
Australia	0.70	0.83	0.75	0.56	1			
Canada	0.94	0.93	0.87	0.51	0.81	1		
Switzerland	0.37	0.61	0.57	0.67	0.54	0.52	1	
Germany	0.75	0.82	0.82	0.53	0.58	0.83	0.71	1

Correlation of equity prices (1979–2011)

	US	UK	Japan	China	Australia	Canada	Switzerland	Germany
US	1							
UK	0.82	1						
Japan	0.53	0.58	1					
China	–	–	–	1				
Australia	0.69	0.70	0.46	–	1			
Canada	0.84	0.73	0.56	–	0.74	1		
Switzerland	0.79	0.80	0.50	–	0.68	0.70	1	
Germany	0.72	0.74	0.50	–	0.61	0.68	0.84	1

Which channels of transmission?

* Common observed global shocks (such as changes in oil prices)
* Global unobserved factors (such as the diffusion of technological progress)
* Specific national or sectorial shocks

What is a GVAR model?

- A GVAR (Global Vector AutoRegressive) model is a global model that combines individual country-specific models in which domestic variables are related to country-specific foreign variables in a consistent manner
- The latter are constructed from the domestic variables so as to match the relative importance of the rest of the world for the country under consideration
- Main ingredients of a GVAR
  - N countries (or regions, states, firms,...)
  - A vector $\mathbf{x}_{it}$ of macroeconomic time series per country i
  - Matrix $\mathbf{W}_i$ of weights capturing the importance of the remaining $N-1$ countries for country i

Why do we need a GVAR model

- Before we start... why do we need GVARs? Couldn't we simply use a standard VAR?
- One known issue of VARs is that they are over-parametrized
- This is often a problem since we have too few observations to properly estimate all parameters in our model
- The GVAR allows to address this "curse of dimensionality" problem

The curse of dimensionality

* Consider as an example a (3×1) vector of time series $\mathbf{x}_{it}$ for country i

$$\mathbf{x}_{it} = \begin{bmatrix} \Delta y_{i1} & \Delta y_{i2} & \dots & \Delta y_{iT} \\ \pi_{i1} & \pi_{i2} & \dots & \pi_{iT} \\ r_{i1} & r_{i2} & \dots & r_{iT} \end{bmatrix} \quad i = 1, \dots, 30$$

* In a globalized world it is likely that core domestic macroeconomic variables are correlated across countries (think of tapering and the relation between r_{US} and r_{UK}...)
* There is scope for modelling jointly the $\mathbf{x}_{it}$

$$
\mathbf{x}_t = \begin{bmatrix} y_{CH,t} \\ \pi_{CH,t} \\ r_{CH,t} \\ --- \\ y_{UK,t} \\ \pi_{UK,t} \\ r_{UK,t} \\ --- \\ \vdots \\ --- \\ y_{US,t} \\ \pi_{US,t} \\ r_{US,t} \end{bmatrix}
$$

- In a standard framework we would set up the following reduced-form VAR

$$\mathbf{x}_t = \mathbf{F}\mathbf{x}_{t-1} + \mathbf{u}_t$$

- Nothing wrong with that... but the fact that we have to estimate 8100 parameters (90 per equation) with –typically– a quarterly sample going from 1980 to 2013 ($\simeq 130$ observations)

The GVAR addresses the curse of dimensionality

- These notes will show how the GVAR addresses the curse of dimensionality...
- ... and how to get to a reduced form VAR of the type

$$\mathbf{x}_t = \mathbf{F}\mathbf{x}_{t-1} + \mathbf{u}_t$$

where

- $\mathbf{x}_t$ is a $\sim (200 \times 1)$ vector of endogenous variables
- $\mathbf{F}$ is a $\sim (200 \times 200)$ matrix of coefficients
- $\mathbf{u}_t$ is a $\sim (200 \times 1)$ vector of reduced-form residuals

Ingredients of the GVAR – A simple example

Let's start with a very simple example

- World is composed by 3 countries: US, UK, and China
- Endogenous domestic variables ($\mathbf{x}_{it}$) are just output and inflation

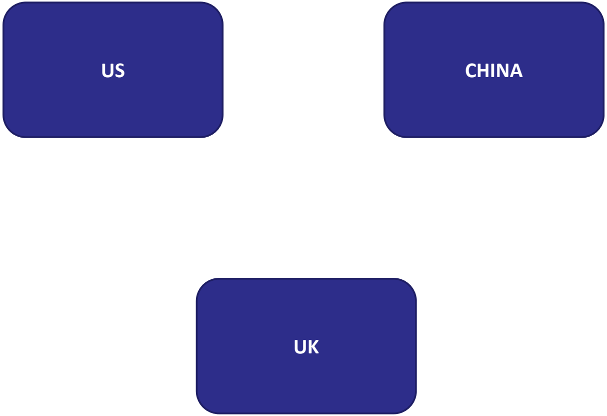
$$\mathbf{x}_{it} = \begin{pmatrix} y_{it} \\ \pi_{it} \end{pmatrix}, \qquad i = CH,\ UK,\ US$$

- We consider trade flows to construct the weights ($\mathbf{W}_i$)

$$\underbrace{\mathbf{EX}_i^j}_{\text{Country } i\text{'s exports to country } j} \qquad \underbrace{\mathbf{IM}_i^j}_{\text{Country } i\text{'s imports from country } j}$$

Let's start with a very simple example

- Assume the world is composed by 3 countries: US, UK, and China



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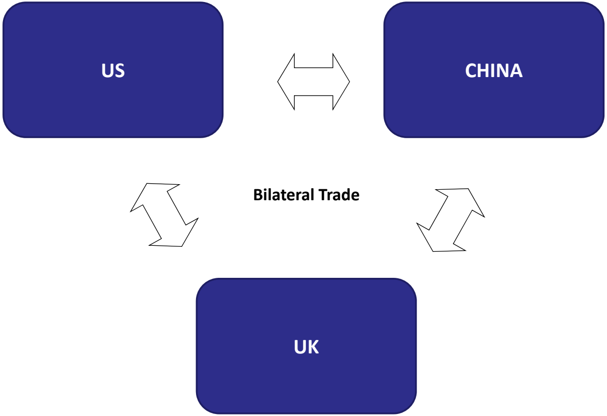

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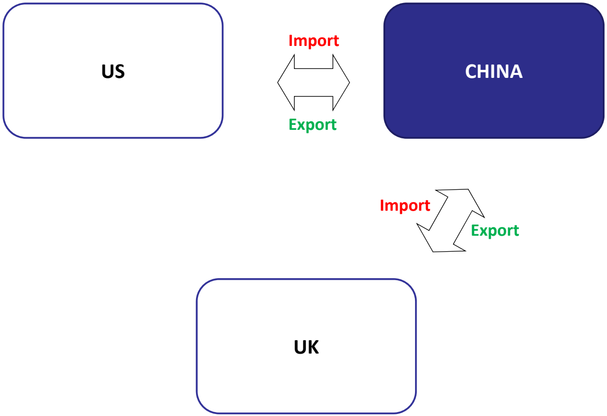
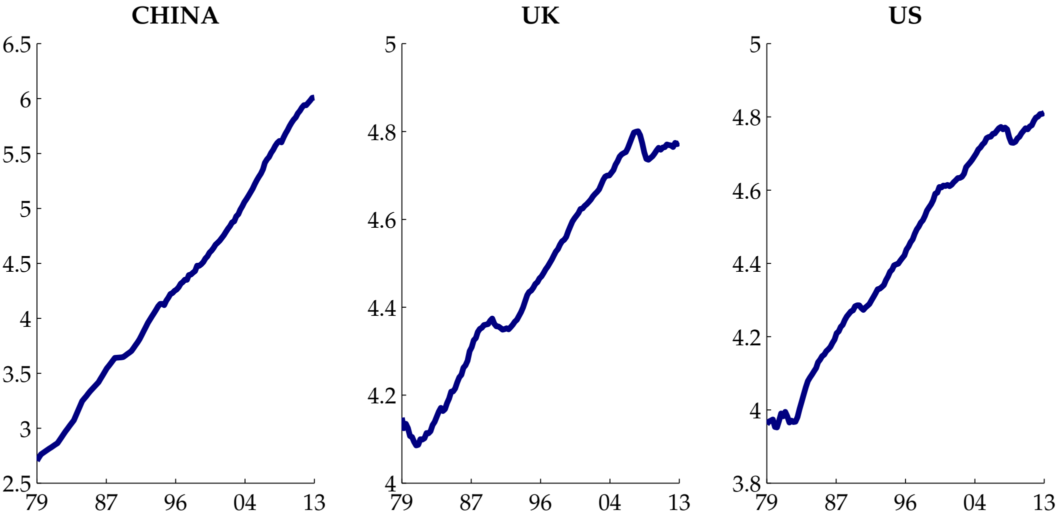
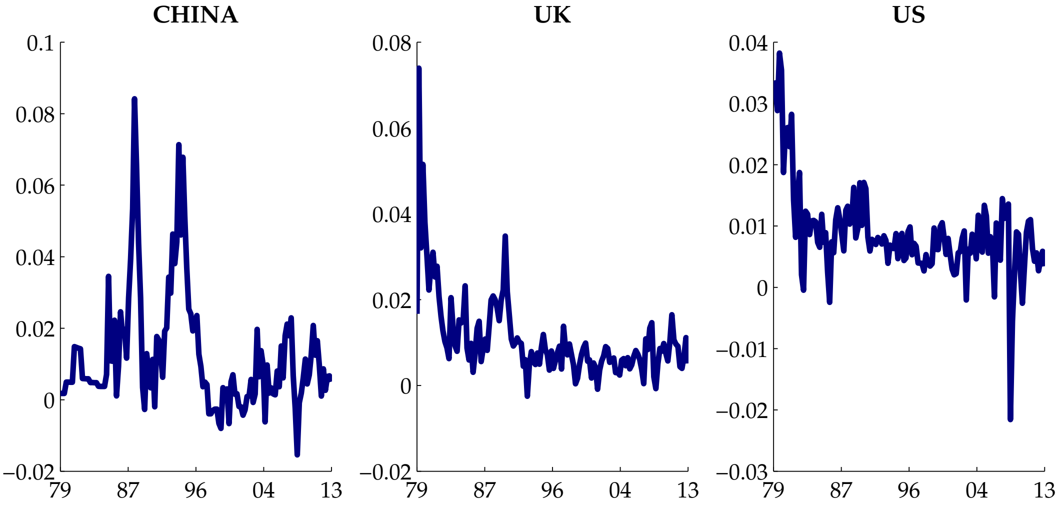


Domestic variables - Real GDP (logs)



Domestic variables - Inflation



Foreign variables

- Foreign variables ($\mathbf{x}_{it}^*$) are a crucial feature of the GVAR
  - Allow inter-linkages between each of the economies and the rest of the world
  - Proxy unobserved global factors
- Computed as cross-sectional averages with weights $\mathbf{W}_{ij}$

$$\mathbf{x}_{it}^* = \sum_{j=0}^{N} \mathbf{W}_{ij}\mathbf{x}_{jt} = \mathbf{W}_i\mathbf{x}_t$$

  - $\mathbf{x}_t = (\mathbf{x}'_{CH,t}, \mathbf{x}'_{UK,t}, \mathbf{x}'_{US,t})'$ is the vector of all endogenous variables
  - $\mathbf{W}_i = (\mathbf{W}_{CH}, \mathbf{W}_{UK}, \mathbf{W}_{US})$ is the country-specific matrix of weights or link matrix

Foreign variables & Link matrix

- In our simple example we would have

$$\mathbf{W}_{CH} = \begin{pmatrix} 0 & 0 & w_{01} & 0 & w_{02} & 0 \\ 0 & 0 & 0 & w_{01} & 0 & w_{02} \end{pmatrix} \qquad \mathbf{x}_t = \begin{pmatrix} y_{CH,t} \\ \pi_{CH,t} \\ --- \\ y_{UK,t} \\ \pi_{UK,t} \\ --- \\ y_{US,t} \\ \pi_{US,t} \end{pmatrix}$$

- Which implies that

$$\mathbf{x}^*_{CH,t} = \begin{pmatrix} y^*_{CH,t} \\ \pi^*_{CH,t} \end{pmatrix} = \mathbf{W}_{CH}\mathbf{x}_t$$

The link matrix

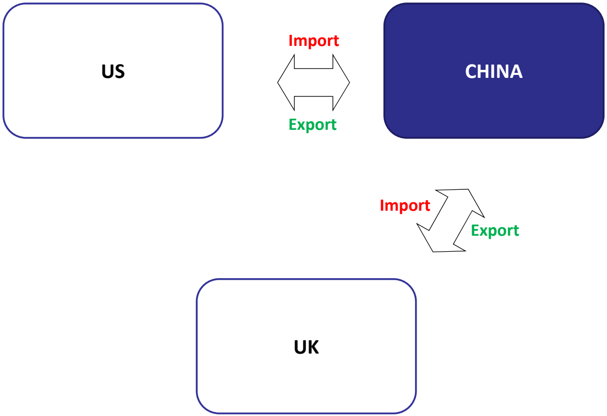
- The link matrix ($\mathbf{W}_i$) captures the importance of country j for country i's economy
- Typically, the weight matrix is trade-based (but can be different)
  - Foreign direct investment, portfolio investment, claims of domestic banks, many others...
- In standard GVAR models the importance of the US for China is computed as

$$\mathbf{w}_{CH,US} = \frac{\mathbf{EX}_{CH}^{US} + \mathbf{IM}_{CH}^{US}}{\mathbf{EX}_{CH} + \mathbf{IM}_{CH}}$$

- Weights can be time-varying to reflect changes in trade structure

Example: China's link matrix

- In our simple example we assume



Example: China's link matrix

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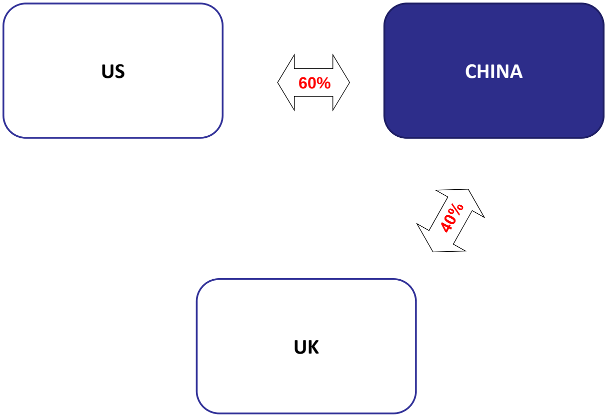


Example: China's link matrix, Domestic & Foreign GDP

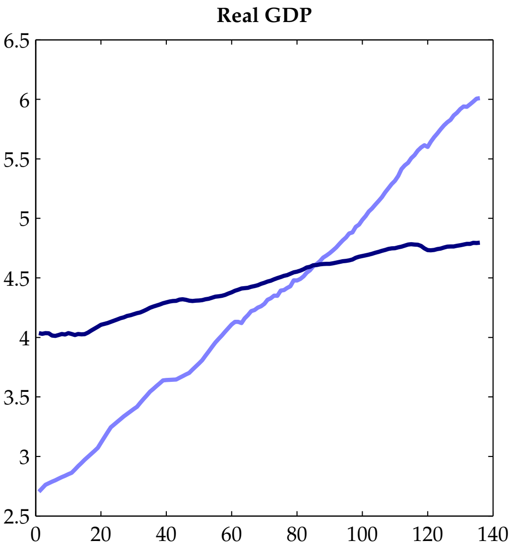
* Which implies

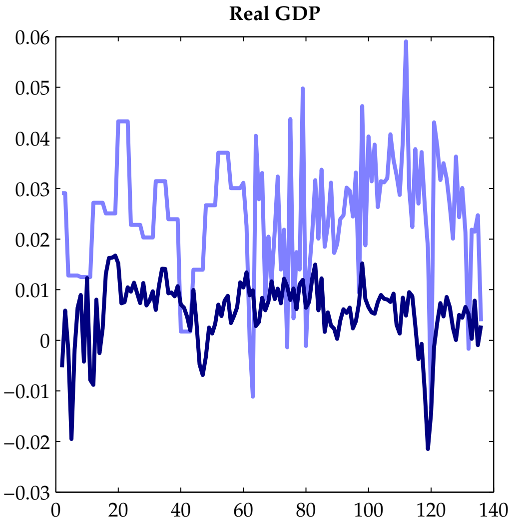
$$\mathbf{W}_{CH} = \begin{pmatrix} 0 & 0 & 0.4 & 0 & 0.6 & 0 \\ 0 & 0 & 0 & 0.4 & 0 & 0.6 \end{pmatrix}$$

* So that foreign GDP for China is

$$y^*_{CH} = 0.4 \cdot y_{UK} + 0.6 \cdot y_{US}$$

Example: China's link matrix, Domestic & Foreign GDP





GVAR modelling strategy

Combining the ingredients

- The ingredients of our simple GVAR are
  - $N = 3$ countries
  - Country-specific (2×1) vectors of domestic $\mathbf{x}_{it}$ and foreign $\mathbf{x}_{it}^*$ variables
  - Global (6×1) vector $\mathbf{x}_t$ of variables
  - Country-specific (2×6) matrices $\mathbf{W}_i$ of weights
- The GVAR modelling approach consists in combining the ingredients through 2 steps
  1. Estimation of country-specific VARX
  2. Combination of the country-specific VARX into a global model

Step 1 – Country-specific VARX models

- Each country is modelled as a country-specific VAR augmented with the foreign variables (VARX)
- VARX(1,1) is

$$\mathbf{x}_{it} = \mathbf{\Phi}\mathbf{x}_{i,t-1} + \mathbf{\Lambda}_{i0}\mathbf{x}^{*}_{it} + \mathbf{\Lambda}_{i1}\mathbf{x}^{*}_{i,t-1} + \mathbf{u}_{it}$$

- In our example, China's model would look like

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \mathbf{\Phi} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \end{bmatrix} + \mathbf{\Lambda}_{i0} \begin{bmatrix} y^{*}_{t} \\ \pi^{*}_{t} \end{bmatrix} + \mathbf{\Lambda}_{i1} \begin{bmatrix} y^{*}_{t-1} \\ \pi^{*}_{t-1} \end{bmatrix} + \begin{bmatrix} u_{yt} \\ u_{\pi t} \end{bmatrix}$$

Country-specific VECMX models

* The above VARX(1,1) model is then written in error correction form

$$\mathbf{x}_{it} - \mathbf{x}_{i,t-1} + \mathbf{x}_{i,t-1} = \mathbf{\Phi}_i \mathbf{x}_{i,t-1} + \mathbf{\Lambda}_{io} \mathbf{x}^*_{it} - \mathbf{\Lambda}_{io} \mathbf{x}^*_{i,t-1} + \mathbf{\Lambda}_{io} \mathbf{x}^*_{i,t-1} + \mathbf{\Lambda}_{i1} \mathbf{x}^*_{i,t-1} + \mathbf{u}_{it}$$

$$\Delta \mathbf{x}_{it} = -(\mathbf{I} - \mathbf{\Phi}_i) \mathbf{x}_{i,t-1} + (\mathbf{\Lambda}_{io} + \mathbf{\Lambda}_{i1}) \mathbf{x}^*_{i,t-1} + \mathbf{\Lambda}_{io} \Delta \mathbf{x}^*_{it}$$

$$\Delta \mathbf{x}_{it} = \alpha_i \left(\beta'_{ix} \mathbf{x}_{i,t-1} + \beta'_{ix^*} \mathbf{x}^*_{i,t-1} \right) + \mathbf{\Lambda}_{io} \Delta \mathbf{x}^*_{it}$$

$$\Delta \mathbf{x}_{it} = \alpha_i \mathbf{ECM}_{i,t-1} + \mathbf{\Lambda}_{io} \Delta \mathbf{x}^*_{it}$$

* Where

$$\mathbf{ECM}_{i,t-1} = \beta'_{ix} \mathbf{x}_{i,t-1} + \beta'_{ix^*} \mathbf{x}^*_{i,t-1}$$

is the vector of long-run cointegrating relations, also known as error-correction terms

[Back to basics] How to estimate VECM models?

- The estimation of $\beta_i = (\beta'_{ix}, \beta'_{ix^*})'$ and of the other parameters is carried out in two separate steps
- Step 1
  - The rank of $\alpha\beta'_i$ is determined using the maximum eigenvalue or the trace statistics
  - β'_i is estimated by imposing suitable exact or possibly over-identifying restrictions on the elements of β'_i
- Step 2
  - Conditional on a given estimate of β'_i, the remaining parameters of the VECMX model are consistently estimated by OLS regressions of the following equation

$$\Delta \mathbf{x}_{it} = \alpha_i ECM_{i,t-1} + \boldsymbol{\Lambda}_{io} \Delta \mathbf{x}^*_{it}$$

    where $\mathbf{ECM}_{i,t-1}$ are the error correction terms corresponding to the r_i cointegrating relations of the i^{th} country model

China's GDP equation in our simple GVAR

- The estimation of a VECMX(1,1) for China yields something like

$$\Delta y_t = \underbrace{\eta_{11} y_{t-1} + \eta_{12} \pi_{t-1}}_{\text{domestic variables in levels}} + \underbrace{\eta_{13} y^*_{t-1} + \eta_{14} \pi^*_{t-1}}_{\text{foreign variables in levels}} + \underbrace{\lambda_{11} \Delta y^*_t + \lambda_{12} \Delta \pi^*_t}_{\text{foreign variables in differences}} + u_{yt}$$

- Note that we allow for cointegration both within the domestic variables and between domestic and foreign variables
- Estimation is performed with reduced rank regression
- Over-identifying restrictions can be imposed

GVAR as a global model of small open economies

- The main assumption underlying the GVAR estimation strategy is a "small open economy assumption"
- In order to be able to include the foreign variables in the models, they have to be (weakly) exogenous
- If not, you run into troubles

Weak exogeneity

- Technically we need to verify the weak exogeneity of $\mathbf{x}_{it}^*$ with respect to the domestic variables
  - "No long-run feedbacks" from $\mathbf{x}_{it}$ to $\mathbf{x}_{it}^*$ (without necessarily ruling out lagged short run feedbacks between the two sets of variable)
- For example, can we safely assume that equity prices in the US do not affect global equity prices?
- Luckily weak exogeneity can be tested!

[Back to basics] How to test for weak exogeneity

* Run the following auxiliary regression for each l^{th} element of the foreign variables $\mathbf{x}_{it}^*$

$$\Delta x_{it,l}^* = \mu_{il} + \sum_{j=1}^{r_i} \gamma_{ij,l} ECM_{i,t-1}^{j} + \sum_{k=1}^{s_i} \varphi_{ik,l} \Delta x_{i,t-k} + \sum_{m=1}^{n_i} \vartheta_{im,l} \Delta \tilde{x}_{i,t-m}^* + \varepsilon_{it,l}$$

* Test the joint significance of the coefficients on the estimated error correction terms

$$H_0 : \gamma_{ij,l} = 0 \quad for\ j = 1, 2, \ldots, r_i$$

* If the F-test cannot reject the null hypothesis, the weak exogeneity assumption is satisfied

Weak exogeneity and VARX specifications

* Weak exogeneity test guides the specification of the country-specific models
* Generally, for foreign GDP and foreign inflation we cannot reject the weak exogeneity assumption
* However, standard specification of the US model does not include
  * Foreign equity prices
  * Foreign short-term and long-term interest rates

Step 2 – Combining the VARX into a global model

* After the country-by-country estimation of the VECMX we can proceed to the second step of the GVAR modelling strategy
  1. Recover the parameters of the VARX models
  2. Combine the VARX into a global model
* The resulting model will have the form of a standard VAR where all variables will be "endogenous"
* This is a purely mechanical step: no estimation is involved!

Selection matrix

- Define a selection matrix that picks up country specific variables from the global vector of endogenous variables

$$\mathbf{S}_{CH} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \mathbf{x}_t = \begin{pmatrix} y_{CH,t} \\ \pi_{CH,t} \\ y_{EA,t} \\ \pi_{EA,t} \\ y_{US,t} \\ \pi_{US,t} \end{pmatrix}$$

- Which implies that

$$\mathbf{x}_{CH,t} = \begin{pmatrix} y_{CH,t} \\ \pi_{CH,t} \end{pmatrix} = \mathbf{S}_{CH}\mathbf{x}_t$$

Re-arrange the VARX models

- VARX(1,1)

$$\mathbf{x}_{it} = \mathbf{\Phi}\mathbf{x}_{i,t-1} + \mathbf{\Lambda}_{i0}\mathbf{x}^*_{it} + \mathbf{\Lambda}_{i1}\mathbf{x}^*_{i,t-1} + \mathbf{u}_{it}$$

- Use the link matrix ($\mathbf{W}_i$) and the selection matrix ($\mathbf{S}_i$)

$$\mathbf{S}_i\mathbf{x}_t = \mathbf{\Phi}\mathbf{S}_i\mathbf{x}_{t-1} + \mathbf{\Lambda}_{i0}\mathbf{W}_i\mathbf{x}_t + \mathbf{\Lambda}_{i1}\mathbf{W}_i\mathbf{x}_{t-1} + \mathbf{u}_{it}$$

- Re-arrange

$$(\mathbf{S}_i - \mathbf{\Lambda}_{i0}\mathbf{W}_i)\,\mathbf{x}_t = (\mathbf{\Phi}\mathbf{S}_i + \mathbf{\Lambda}_{i1}\mathbf{W}_i)\,\mathbf{x}_{t-1} + \mathbf{u}_{it}$$

- Re-name

$$\mathbf{G}_i\mathbf{x}_t = \mathbf{H}_i\mathbf{x}_{t-1} + \mathbf{u}_{it}$$

The global model

- Stack all country-specific models

$$\begin{bmatrix} \mathbf{G}_{CH} \\ \mathbf{G}_{UK} \\ \mathbf{G}_{US} \end{bmatrix} \mathbf{x}_t = \begin{bmatrix} \mathbf{H}_{CH} \\ \mathbf{H}_{UK} \\ \mathbf{H}_{US} \end{bmatrix} \mathbf{x}_{t-1} + \begin{bmatrix} \mathbf{u}_{CH,t} \\ \mathbf{u}_{UK,t} \\ \mathbf{u}_{US,t} \end{bmatrix}$$

- To finally get the GVAR

$$\mathbf{G}\mathbf{x}_t = \mathbf{H}\mathbf{x}_{t-1} + \mathbf{u}_t$$

where $\mathbf{G} = (\mathbf{G}'_{CH}, \mathbf{G}'_{UK}, \mathbf{G}'_{US})'$, $\mathbf{H} = (\mathbf{H}'_{CH}, \mathbf{H}'_{UK}, \mathbf{H}'_{US})'$, and $\mathbf{u} = (\mathbf{u}'_{CH}, \mathbf{u}'_{UK}, \mathbf{u}'_{US})'$

VARX models

- In our simple (3 countries, 2 variables) example we have

$$\begin{pmatrix} y_{CH,t} \\ \pi_{CH,t} \end{pmatrix} = \begin{pmatrix} \phi_{0,11} & \phi_{0,12} \\ \phi_{0,21} & \phi_{0,22} \end{pmatrix} \begin{pmatrix} y_{CH,t-1} \\ \pi_{CH,t-1} \end{pmatrix} + \begin{pmatrix} \lambda_{00,11} & \lambda_{00,12} \\ \lambda_{00,21} & \lambda_{00,22} \end{pmatrix} \begin{pmatrix} y^*_{CH,t} \\ \pi^*_{CH,t} \end{pmatrix} + \begin{pmatrix} \lambda_{01,11} & \lambda_{01,12} \\ \lambda_{01,21} & \lambda_{01,22} \end{pmatrix} \begin{pmatrix} y^*_{CH,t-1} \\ \pi^*_{CH,t-1} \end{pmatrix} + \begin{pmatrix} u^y_{CH} \\ u^\pi_{CH} \end{pmatrix}$$

$$\begin{pmatrix} y_{UK,t} \\ \pi_{UK,t} \end{pmatrix} = \begin{pmatrix} \phi_{1,11} & \phi_{1,12} \\ \phi_{1,21} & \phi_{1,22} \end{pmatrix} \begin{pmatrix} y_{UK,t-1} \\ \pi_{UK,t-1} \end{pmatrix} + \begin{pmatrix} \lambda_{10,11} & \lambda_{10,12} \\ \lambda_{10,21} & \lambda_{10,22} \end{pmatrix} \begin{pmatrix} y^*_{UK,t} \\ \pi^*_{UK,t} \end{pmatrix} + \begin{pmatrix} \lambda_{11,11} & \lambda_{11,12} \\ \lambda_{11,21} & \lambda_{11,22} \end{pmatrix} \begin{pmatrix} y^*_{UK,t-1} \\ \pi^*_{UK,t-1} \end{pmatrix} + \begin{pmatrix} u^y_{UK} \\ u^\pi_{UK} \end{pmatrix}$$

$$\begin{pmatrix} y_{US,t} \\ \pi_{US,t} \end{pmatrix} = \begin{pmatrix} \phi_{2,11} & \phi_{2,12} \\ \phi_{2,21} & \phi_{2,22} \end{pmatrix} \begin{pmatrix} y_{US,t-1} \\ \pi_{US,t-1} \end{pmatrix} + \begin{pmatrix} \lambda_{20,11} & \lambda_{20,12} \\ \lambda_{20,21} & \lambda_{20,22} \end{pmatrix} \begin{pmatrix} y^*_{US,t} \\ \pi^*_{US,t} \end{pmatrix} + \begin{pmatrix} \lambda_{21,11} & \lambda_{21,12} \\ \lambda_{21,21} & \lambda_{21,22} \end{pmatrix} \begin{pmatrix} y^*_{US,t-1} \\ \pi^*_{US,t-1} \end{pmatrix} + \begin{pmatrix} u^y_{US} \\ u^\pi_{US} \end{pmatrix}$$

GVAR model

$$
\begin{bmatrix}
1 & 0 & -\hat{\lambda}_{00,11}w_{01} & -\hat{\lambda}_{00,12}w_{01} & -\hat{\lambda}_{00,11}w_{02} & -\hat{\lambda}_{00,12}w_{02} \\
0 & 1 & -\hat{\lambda}_{00,21}w_{01} & -\hat{\lambda}_{00,22}w_{01} & -\hat{\lambda}_{00,21}w_{02} & -\hat{\lambda}_{00,22}w_{02} \\
-\hat{\lambda}_{10,11}w_{10} & -\hat{\lambda}_{10,12}w_{10} & 1 & 0 & -\hat{\lambda}_{10,11}w_{12} & -\hat{\lambda}_{10,12}w_{12} \\
-\hat{\lambda}_{10,21}w_{10} & -\hat{\lambda}_{10,22}w_{10} & 0 & 1 & -\hat{\lambda}_{10,21}w_{12} & -\hat{\lambda}_{10,22}w_{12} \\
-\hat{\lambda}_{20,11}w_{20} & -\hat{\lambda}_{20,12}w_{20} & -\hat{\lambda}_{20,11}w_{21} & -\hat{\lambda}_{20,12}w_{21} & 1 & 0 \\
-\hat{\lambda}_{20,21}w_{20} & -\hat{\lambda}_{20,22}w_{20} & -\hat{\lambda}_{20,21}w_{01} & -\hat{\lambda}_{20,22}w_{21} & 0 & 1
\end{bmatrix}
\begin{bmatrix}
y_{CH,t} \\ \pi_{CH,t} \\ y_{UK,t} \\ \pi_{UK,t} \\ y_{US,t} \\ \pi_{US,t}
\end{bmatrix}
=
$$

$$
=
\begin{bmatrix}
\hat{\phi}_{0,11} & \hat{\phi}_{0,12} & \hat{\lambda}_{01,11}w_{01} & \hat{\lambda}_{01,12}w_{01} & \hat{\lambda}_{01,11}w_{02} & \hat{\lambda}_{01,12}w_{02} \\
\hat{\phi}_{0,21} & \hat{\phi}_{0,22} & \hat{\lambda}_{01,21}w_{01} & \hat{\lambda}_{01,22}w_{01} & \hat{\lambda}_{01,21}w_{02} & \hat{\lambda}_{01,22}w_{02} \\
\hat{\lambda}_{11,11}w_{10} & \hat{\lambda}_{11,12}w_{10} & \hat{\phi}_{1,11} & \hat{\phi}_{1,12} & \hat{\lambda}_{11,11}w_{12} & \hat{\lambda}_{11,12}w_{12} \\
\hat{\lambda}_{11,21}w_{10} & \hat{\lambda}_{11,22}w_{10} & \hat{\phi}_{1,21} & \hat{\phi}_{1,22} & \hat{\lambda}_{11,21}w_{12} & \hat{\lambda}_{11,22}w_{12} \\
\hat{\lambda}_{21,11}w_{20} & \hat{\lambda}_{21,12}w_{20} & \hat{\lambda}_{21,11}w_{21} & \hat{\lambda}_{21,12}w_{21} & \hat{\phi}_{2,11} & \hat{\phi}_{2,12} \\
\hat{\lambda}_{21,21}w_{20} & \hat{\lambda}_{21,22}w_{20} & \hat{\lambda}_{21,21}w_{01} & \hat{\lambda}_{21,22}w_{21} & \hat{\phi}_{2,21} & \hat{\phi}_{2,22}
\end{bmatrix}
\begin{bmatrix}
y_{CH,t-1} \\ \pi_{CH,t-1} \\ y_{UK,t-1} \\ \pi_{UK,t-1} \\ y_{US,t-1} \\ \pi_{US,t-1}
\end{bmatrix}
+
\begin{bmatrix}
u^{y}_{CH} \\ u^{\pi}_{CH} \\ u^{y}_{UK} \\ u^{\pi}_{UK} \\ u^{y}_{US} \\ u^{\pi}_{US}
\end{bmatrix}
$$

- The GVAR in the previous page has the familiar form of a VAR(1) with 6 variables

$$\mathbf{G}\mathbf{x}_t = \mathbf{H}\mathbf{x}_{t-1} + \mathbf{u}_t$$

- Or of a reduced-form VAR(1) if we premultiply by $\mathbf{G}^{-1}$

$$\mathbf{x}_t = \mathcal{F}\mathbf{x}_{t-1} + \xi_t$$

  where $\mathcal{F} = \mathbf{G}^{-1}\mathbf{H}$ and $\xi_t = \mathbf{G}^{-1}\mathbf{u}_t$

- An important difference: the errors $\mathbf{u}_t$ are not "structural"

The GVAR variance-covariance matrix

The GVAR variance-covariance matrix

- The residuals of the GVAR are

$$\mathbf{u}_t = \left[\begin{array}{cccccc} u_{CH}^{y\prime} & u_{CH}^{\pi\prime} & u_{UK}^{y\prime} & u_{UK}^{\pi\prime} & u_{US}^{y\prime} & u_{US}^{\pi\prime} \end{array}\right]'$$

- In our example the elements of $\mathbf{u}_t$ are exactly the reduced-form residuals we obtained in the country specific VECMX estimation

- As in normal VARs, the residuals of the country-specific models may be correlated
  - For example we should expect u_{CH}^{y} to be correlated with u_{CH}^{π}

- How about the correlations of the residuals across countries?
  - For example, what would be the correlation between the China GDP residuals (u_{CH}^{y}) and the US GDP residuals (u_{US}^{y})?

GVAR variance-covariance matrix & Global shocks

- To understand the residuals cross-country correlation, let's make the following example
  - Estimate a VAR for the US $\left(y'_{US}, \pi'_{US}\right)'$ and back out the GDP residuals
  - Estimate a VAR for the UK $\left(y'_{UK}, \pi'_{UK}\right)'$ and back out the GDP residuals
  - What do you expect to see in the US and UK GDP residuals in 2008Q4?
- In the presence of global shocks or international spillovers we expect a positive correlation between the residuals across countries
- But the foreign variables will pick up some of these co-movements $\Longrightarrow$ we expect the $\mathbf{x}^*_{it}$ to lower the correlation

The GVAR variance-covariance matrix

$$\begin{bmatrix} VAR(u^y_{CH}) & COV(u^y_{CH}, u^\pi_{CH}) & COV(u^y_{CH}, u^y_{UK}) & COV(u^y_{CH}, u^\pi_{UK}) & COV(u^y_{CH}, u^y_{US}) & COV(u^y_{CH}, u^\pi_{US}) \\ - & VAR(u^\pi_{CH}) & COV(u^\pi_{CH}, u^y_{UK}) & COV(u^\pi_{CH}, u^\pi_{UK}) & COV(u^\pi_{CH}, u^y_{US}) & COV(u^\pi_{CH}, u^\pi_{US}) \\ - & - & VAR(u^y_{UK}) & COV(u^y_{UK}, u^\pi_{UK}) & COV(u^y_{UK}, u^y_{US}) & COV(u^y_{UK}, u^\pi_{US}) \\ - & - & - & VAR(u^\pi_{UK}) & COV(u^\pi_{UK}, u^y_{US}) & COV(u^\pi_{UK}, u^\pi_{US}) \\ - & - & - & - & VAR(u^y_{US}) & COV(u^y_{US}, u^\pi_{US}) \\ - & - & - & - & - & VAR(u^\pi_{US}) \end{bmatrix}$$

* Can be re-written more compactly as

$$\boldsymbol{\Sigma} = \begin{bmatrix} \Sigma_{CH} & \Sigma_{CH,UK} & \Sigma_{CH,US} \\ - & \Sigma_{UK} & \Sigma_{UK,US} \\ - & - & \Sigma_{US} \end{bmatrix}$$

Dynamic Analysis

How do country-specific shocks propagate internationally?

- We constructed a global model which has a VAR(1) representation
- This can be now used to answer interesting questions...
- For example
  - What is the effect of a monetary policy tightening in the US on UK GDP?
  - How does a slowdown in China affect UK inflation?
- The GVAR can be used to perform standard dynamic analysis (impulse responses analyses, forecasting,...)

How about the identification of shocks?

- We know from the VAR literature that, when the reduced-form error terms are correlated we cannot interpret them as structural shocks
- In the GVAR the issue is slightly more complicated than that... we need to take into account 2 types of correlations
  1. The country-specific residuals are correlated (i.e., Σ_{CH} is not diagonal)
     - We need to identify the shocks within the country systems!
  2. The cross-country residuals are correlated (i.e., $\Sigma_{CH,US}$ is not zero)
     - Assume that you want to simulate a shock to the US GDP residual u^y_{US}
     - If the shock is correlated with the China GDP residual, then u^y_{CH} will also increase
     - Reverse causality issues

Dealing with the cross-country correlation of the residuals

* By “conditioning” the domestic variables on weakly exogenous foreign variables (proxies common unobserved global factors) we reduce the degree of correlation of the residuals across countries/regions ⇒ weak cross-sectional dependence

* “Cleaning” the residuals from the effect of global factors allows us to interpret the shock internationally

Foreign variables & GVAR variance-covariance matrix

- VAR: $\mathbf{x}_{it} = \mathbf{\Phi}\mathbf{x}_{i,t-1} + \mathbf{u}_{it}$
- VARX: $\mathbf{x}_{it} = \mathbf{\Phi}\mathbf{x}_{i,t-1} + \mathbf{\Lambda}_{i0}\mathbf{x}^*_{it} + \mathbf{\Lambda}_{i1}\mathbf{x}^*_{i,t-1} + \mathbf{u}_{it}$

VAR

$$\mathbf{\Sigma} = \begin{bmatrix} \Sigma_{CH} & \Sigma_{CH,UK} & \Sigma_{CH,US} \\ - & \Sigma_{UK} & \Sigma_{UK,US} \\ - & - & \Sigma_{US} \end{bmatrix}$$

VARX

$$\mathbf{\Sigma} = \begin{bmatrix} \Sigma_{CH} & \simeq 0 & \simeq 0 \\ - & \Sigma_{UK} & \simeq 0 \\ - & - & \Sigma_{US} \end{bmatrix}$$

Dealing with the country-specific correlation of the residuals

* If the residuals are cross-sectionally weakly correlated we can apply standard identification schemes to the country-specific VARX
* For example
    * Aggregate shocks
    * Monetary policy shocks
    * Credit supply shocks
    * Housing demand
    * ...

  obtained with Cholesky, sign restrictions,...

GVAR summary

1. First step: country-specific VARX models
   * Construct the foreign variables
   * Estimate the parameters of the VECMX models
2. Second step: global model
   * Re-arrange the country specific VARX
   * Combine them into a global model
3. Dynamic analysis
   * Identify a shock of interest
   * Compute impulse responses

The GVAR Toolbox

The GVAR Toolbox

* The GVAR Toolbox is a collection of MatLab procedures with an Excel-based interface designed for the purpose of GVAR modelling
* It is an accessible and easy-to-use package (no background knowledge of MatLab required!)
* Available at Vanessa Smith's page
  * https://sites.google.com/site/gvarmodelling/gvar-toolbox
* Includes a very detailed user guide (with many details on the econometrics)

Standard GVAR model – 33 countries

United States	**Euro Area**	**Latin America**
China	Germany	Brazil
Japan	France	Mexico
United Kingdom	Italy	Argentina
	Spain	Chile
Canada	Netherlands	Peru
Australia	Belgium	
New Zealand	Austria	
	Finland	
Rest of Asia	**Rest of W. Europe**	**Rest of the World**
Korea	Sweden	India
Indonesia	Switzerland	South Africa
Thailand	Norway	Turkey
Philippines		Saudi Arabia
Malaysia		
Singapore		

Standard GVAR model – 7 variables

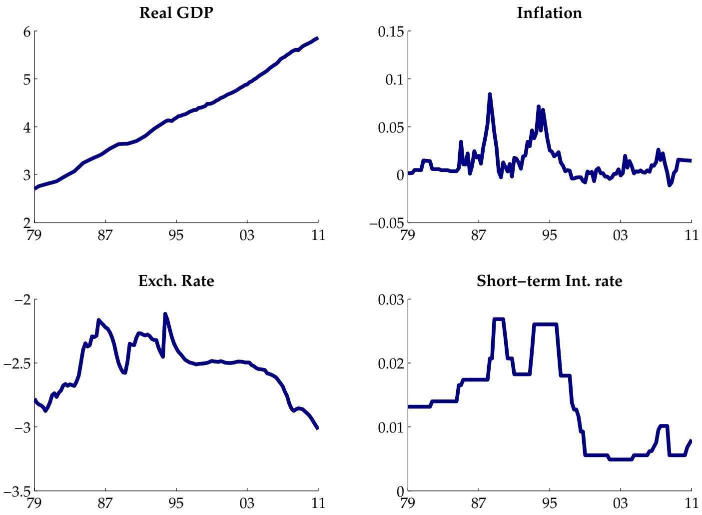
* 6 endogenous variables

Variable	**Measure**
Real GDP (y_{it})	$\log(GDP_{it}/CPI_{it})$
CPI Inflation (π_{it})	$\log(CPI_{it}/CPI_{it-1})$
Real Equity Price (q_{it})	$\log(EQ_{it}/CPI_{it})$
Real Exchange Rate ($e - p_{it}$)	$\log(E_{it}/CPI_{it})$
Short-term Interest Rate (ρ^S_{it})	$0.25 \cdot ln(1 + R^S_{it}/100)$
Long-term Interest Rate (ρ^L_{it})	$0.25 \cdot ln(1 + R^L_{it}/100)$

* 1 global variable

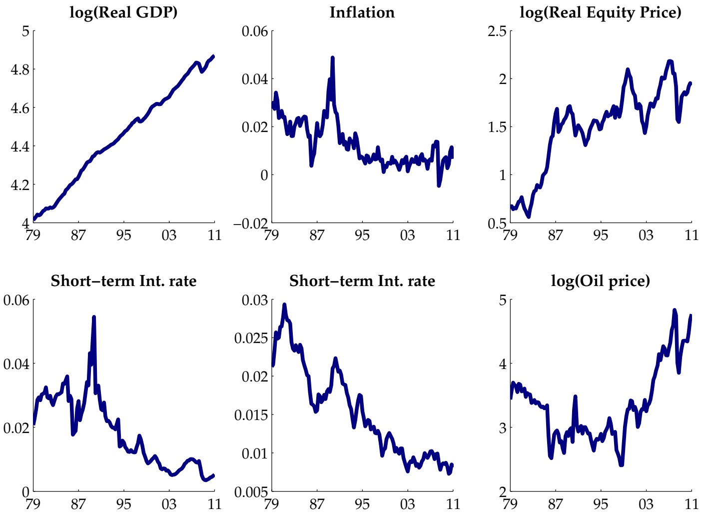
Variable	**Measure**
Oil price (p^{oil}_t)	$\log(OIL_t)$

China's domestic variables in the standard GVAR



* Chinese equity prices and long-term government bond data are not available from 1979
* GVAR allows for missing data in country specific models...
* but doesn't allow for unbalanced data

China's foreign variables in the standard GVAR



- Even if some country-specific domestic variables are missing, all foreign variables can be included in the VARX
- Foreign exchange rates are included as a foreign variable only for the US
- Oil price: really exogenous? (Addressed in next release of the GVAR Toolbox)

China's VARX model in the standard GVAR

- China's VARX would look like

$$\begin{bmatrix} y_t \\ \pi_t \\ (e-p)_t \\ \rho^S_t \end{bmatrix} = \mathbf{\Phi} \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ (e-p)_{t-1} \\ \rho^S_{t-1} \end{bmatrix} + \mathbf{\Lambda}_{i0} \begin{bmatrix} y^*_t \\ \pi^*_t \\ q^*_t \\ \rho^{S*}_t \\ \rho^{L*}_t \end{bmatrix} + \mathbf{\Lambda}_{i1} \begin{bmatrix} y^*_{t-1} \\ \pi^*_{t-1} \\ q^*_{t-1} \\ \rho^{S*}_{t-1} \\ \rho^{L*}_{t-1} \end{bmatrix} + \mathbf{\Xi} p^{oil} + \begin{bmatrix} u_{y,t} \\ u_{\pi,t} \\ u_{(e-p),t} \\ u_{\rho^S,t} \end{bmatrix}$$

Standard GVAR model – Data sets

- First GVAR release 1979.I–2003.IV $\Longrightarrow$ 99 observations
- Last GVAR release 1979.I–2013.I $\Longrightarrow$ 136 observations
- A country-specific VARX(2,1) may require the estimation of up to 156 parameters (26 per equation)
- What can we do with more data?
  - Estimate richer models (a VARX(4,2) for example)
  - Reduce the sample period (say 1985-2013) to cut hyperinflation periods / structural breaks or to include new countries (for which data availability is more limited)

Standard GVAR model – Weights

* Annual trade flows from IMF DOTS
* For example in 2009

Year and trading bloc	United States	Euro area	Japan	China	Latin America
United States	-	0.17	0.18	0.22	0.51
Euro area	0.15	-	0.11	0.18	0.15
Japan	0.07	0.05	-	0.15	0.04
China	0.18	0.15	0.26	-	0.12
Latin America	0.18	0.06	0.03	0.05	-
Rest of the world	0.42	0.58	0.42	0.39	0.18
Total	1	1	1	1	1

Examples

Examples of GVAR applications

- US housing demand shocks
  - Cesa-Bianchi (2013). “Housing cycles and macroeconomic fluctuations: A global perspective”, Journal of International Money and Finance
- China GDP shocks
  - Cesa-Bianchi, Pesaran, Rebucci, Xu (2012). “China’s Emergence in the World Economy and Business Cycles in Latin America”, Economia

Example 1. International transmission of US housing demand shocks

- Cesa-Bianchi (2013). "Housing cycles and macroeconomic fluctuations: A global perspective", Journal of International Money and Finance
- Questions
  - How are housing demand shocks transmitted to the real economy?
  - What is the impact of a US housing demand shock on domestic and global GDP?

Details on the GVAR specification

- Estimation period: 1983Q1 to 2009Q4
- 33 country–specific VARX models
- Country-specific models include the following endogenous

Variable	**Measure**
Real GDP (y_{it})	$\log(GDP_{it}/CPI_{it})$
CPI Inflation (π_{it})	$\log(CPI_{it}/CPI_{it-1})$
Real House Price (hp_{it})	$\log(HP_{it}/CPI_{it})$
Real Equity Price (q_{it})	$\log(EQ_{it}/CPI_{it})$
Real Exchange Rate ($e-p_{it}$)	$\log(E_{it}/CPI_{it})$
Short-term Interest Rate (ρ^S_{it})	$0.25 \cdot ln(1+R^S_{it}/100)$
Long-term Interest Rate (ρ^L_{it})	$0.25 \cdot ln(1+R^L_{it}/100)$

How to identify housing demand shocks?

- Housing demand shock
  - Real house price increase
  - Nominal short-term interest rate does not fall $\longrightarrow$ To rule out expansionary monetary policy shocks
  - No contemporaneous impact on GDP and CPI inflation $\longrightarrow$ To rule out more fundamental expansionary shocks
- Operationally, the identification is achieved with a standard Cholesky decomposition

$$x_{it} = \left(y_i', \pi_i', \rho_i^{S\prime}, hp_i', \rho_i^{L\prime}, (e-p)_i', q_i' \right)'$$

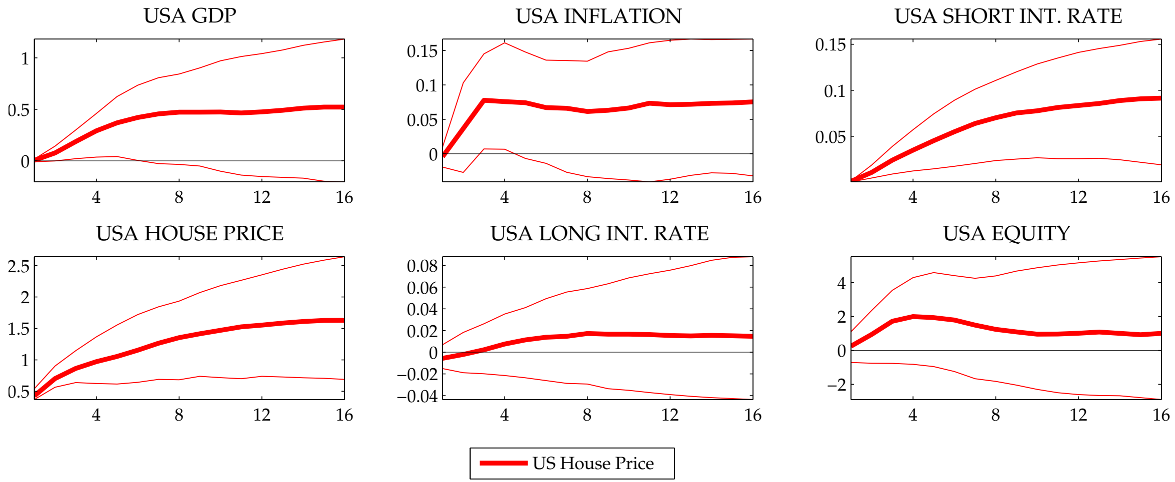
* Let P_0 be lower triangular Cholesky factor of the residuals covariance matrix of country 0, then the GVAR model can be written as

$$\mathbf{G}\mathbf{x}_t = \mathbf{H}\mathbf{x}_{t-1} + \mathbf{P}^G\mathbf{v}_t.$$

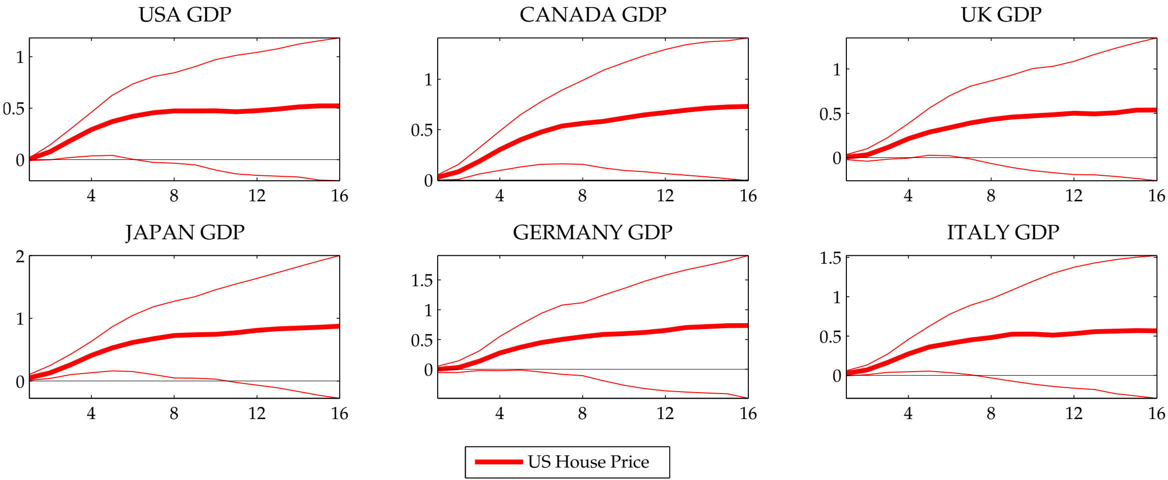
where $\mathbf{P}^G\mathbf{v}_t = \mathbf{u}_t$ and

$$\mathbf{P}^G = \begin{bmatrix} \mathbf{P}_0 & 0 & \cdots & 0 \\ 0 & \mathbf{I}_{k_1} & \cdots & 0 \\ \vdots & \cdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{I}_{k_N} \end{bmatrix}, \quad \mathbf{v}_t = \begin{bmatrix} \mathbf{v}_0 \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_N \end{bmatrix}, \quad \Sigma_v = \begin{bmatrix} \mathbf{I} & \Sigma_{\mathbf{v}_0\mathbf{u}1} & \cdots & \Sigma_{\mathbf{v}_0\mathbf{u}_N} \\ \Sigma_{\mathbf{u}_1\mathbf{v}_0} & \Sigma_{\mathbf{u}_1} & \cdots & \Sigma_{\mathbf{u}_1\mathbf{u}_N} \\ \vdots & \cdots & \ddots & \vdots \\ \Sigma_{\mathbf{u}_N\mathbf{v}_0} & \Sigma_{\mathbf{u}_N\mathbf{u}_1} & \cdots & \Sigma_{\mathbf{u}_N} \end{bmatrix}$$

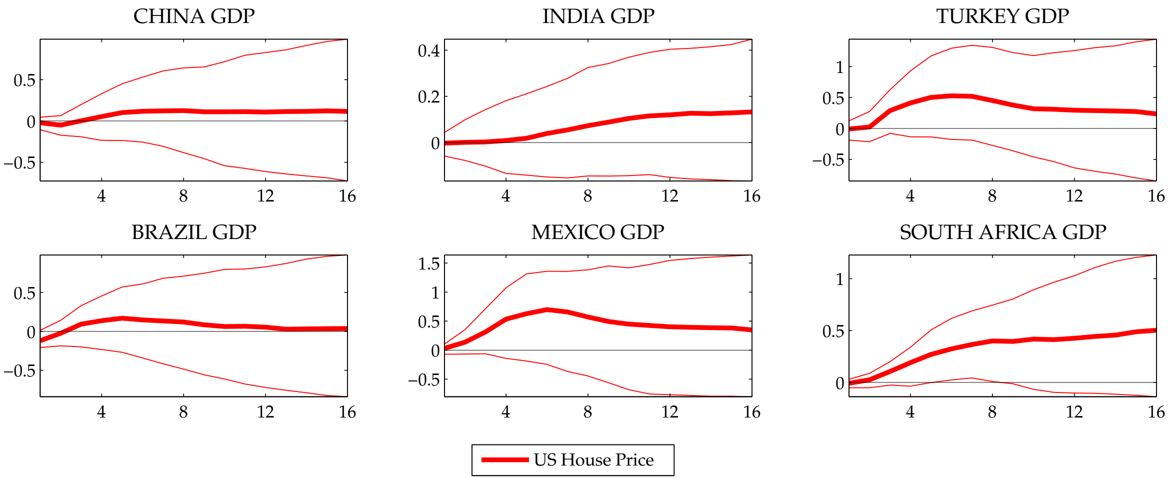
The US housing demand shock leads to an expansion in the US economy



US housing demand shock has significant spillovers on AEs...



...while EMEs response is heterogeneous



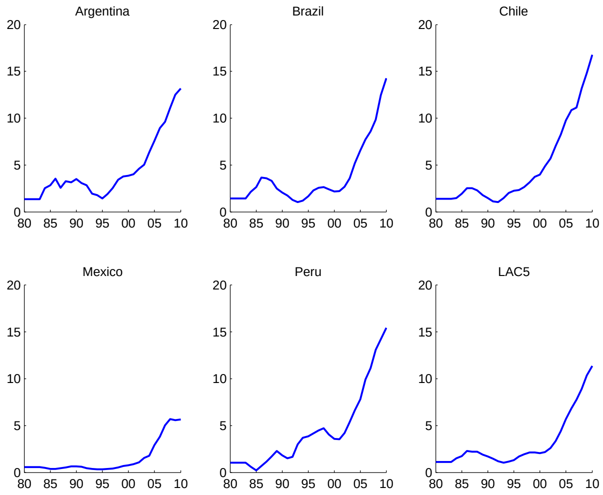
Conclusions and implications

- A US housing demand shock is transmitted to most AEs but has no impact on four large EMEs
- Regionalization hypothesis advanced by Hirata, Kose and Otrok (2011)
  - Business cycle synchronization has increased among AEs and among EMEs separately while the relative importance of the global factor has declined
  - Some EMEs have become more resilient to shocks originated in AEs
- Decreased importance of US shocks in the global economy
  - Rather than decoupling from the world economy, many EMEs shifted their loading from the US into other EMEs

Example 2. International transmission of Chinese GDP shocks

- Cesa-Bianchi, Pesaran, Rebucci, Xu (2012). “China’s Emergence in the World Economy and Business Cycles in Latin America”, Economia
- Questions
  - Has the international transmission of shocks from China changed over time?

The changing role of China



- Chart displays the evolution of trade shares of China in LAC over time
- World economy has undergone profound changes with the emergence of China and other large developing countries
- The transmission mechanisms of the international business cycle to Latin American may have changed

A counterfactual exercise

- Set up an experiment to assess whether the role of China for LAC has changed over time
- First step: estimate the country-specific VARX models with time-varying weights
  - Construct the foreign variables with time varying weights
  - Estimate the VARX as usual
- Second step: use counterfactual sets of weights to simulate the model
  - Years: 1985, 1995, 2005, 2009

GVAR model & Counterfactual weights

- Remember that the GVAR can be written as

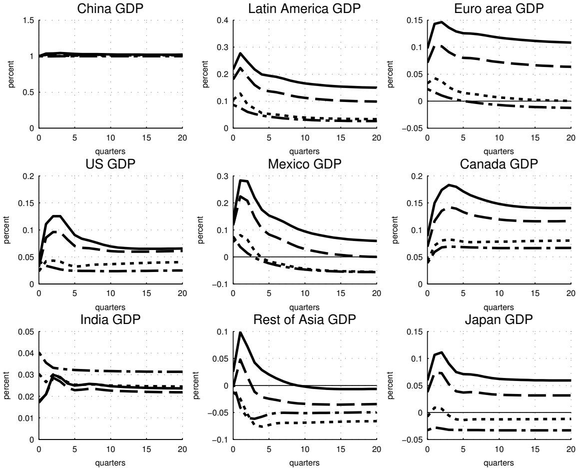
$$\begin{bmatrix} 1 & 0 & -\hat{\lambda}_{00,11}w_{01} & -\hat{\lambda}_{00,12}w_{01} & -\hat{\lambda}_{00,11}w_{02} & -\hat{\lambda}_{00,12}w_{02} \\ 0 & 1 & -\hat{\lambda}_{00,21}w_{01} & -\hat{\lambda}_{00,22}w_{01} & -\hat{\lambda}_{00,21}w_{02} & -\hat{\lambda}_{00,22}w_{02} \\ -\hat{\lambda}_{10,11}w_{10} & -\hat{\lambda}_{10,12}w_{10} & 1 & 0 & -\hat{\lambda}_{10,11}w_{12} & -\hat{\lambda}_{10,12}w_{12} \\ -\hat{\lambda}_{10,21}w_{10} & -\hat{\lambda}_{10,22}w_{10} & 0 & 1 & -\hat{\lambda}_{10,21}w_{12} & -\hat{\lambda}_{10,22}w_{12} \\ -\hat{\lambda}_{20,11}w_{20} & -\hat{\lambda}_{20,12}w_{20} & -\hat{\lambda}_{20,11}w_{21} & -\hat{\lambda}_{20,12}w_{21} & 1 & 0 \\ -\hat{\lambda}_{20,21}w_{20} & -\hat{\lambda}_{20,22}w_{20} & -\hat{\lambda}_{20,21}w_{01} & -\hat{\lambda}_{20,22}w_{21} & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{CH,t} \\ \pi_{CH,t} \\ y_{UK,t} \\ \pi_{UK,t} \\ y_{US,t} \\ \pi_{US,t} \end{bmatrix} = \dots$$

GVAR model & Counterfactual weights

* Impose a set of weights different from the one used in the estimation

$$\begin{bmatrix} 1 & 0 & -\hat{\lambda}_{00,11} w_{01} & -\hat{\lambda}_{00,12} w_{01} & -\hat{\lambda}_{00,11} w_{02} & -\hat{\lambda}_{00,12} w_{02} \\ 0 & 1 & -\hat{\lambda}_{00,21} w_{01} & -\hat{\lambda}_{00,22} w_{01} & -\hat{\lambda}_{00,21} w_{02} & -\hat{\lambda}_{00,22} w_{02} \\ -\hat{\lambda}_{10,11} w_{10} & -\hat{\lambda}_{10,12} w_{10} & 1 & 0 & -\hat{\lambda}_{10,11} w_{12} & -\hat{\lambda}_{10,12} w_{12} \\ -\hat{\lambda}_{10,21} w_{10} & -\hat{\lambda}_{10,22} w_{10} & 0 & 1 & -\hat{\lambda}_{10,21} w_{12} & -\hat{\lambda}_{10,22} w_{12} \\ -\hat{\lambda}_{20,11} w_{20} & -\hat{\lambda}_{20,12} w_{20} & -\hat{\lambda}_{20,11} w_{21} & -\hat{\lambda}_{20,12} w_{21} & 1 & 0 \\ -\hat{\lambda}_{20,21} w_{20} & -\hat{\lambda}_{20,22} w_{20} & -\hat{\lambda}_{20,21} w_{01} & -\hat{\lambda}_{20,22} w_{21} & 0 & 1 \end{bmatrix} \begin{bmatrix} y_{CH,t} \\ \pi_{CH,t} \\ y_{UK,t} \\ \pi_{UK,t} \\ y_{US,t} \\ \pi_{US,t} \end{bmatrix} = \dots$$

The changing transmission of Chinese shocks

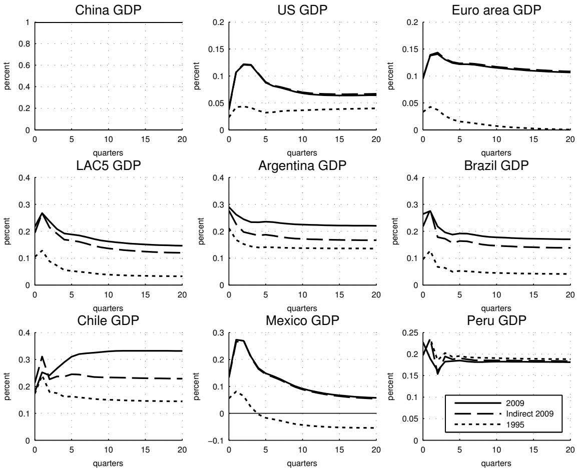


- China GDP shock matters much more with recent weights
- This is true for both AEs and EMEs
- The observed changes in the transmission of the China GDP shock are likely to have played an important role in the unfolding of the recent global crisis (and most importantly of the recovery)

Playing with the link matrix

- Is the increased impact of a Chinese GDP shock on LAC due to direct or indirect trade linkages?
  - Stronger bilateral trade ties between China and LAC?
  - Or to stronger impact from China to LAC's traditional (and largest) trading partners (US, euro area)?
- Counterfactual exercise
  - Take the 2009 link matrix
  - Set the weight of China for LAC economies to its 1995 level ($\simeq 0$)
  - Is the impact of Chinese shock on LAC effect still large? If yes, then it's mostly indirect channel

Direct VS Indirect effects



- We plot the response with 1995 weights, 2009 weights and with the counterfactual link matrix
- We exclude Mexico from the counterfactual since its very large tie to the US would introduce a distortion in the results
- Indirect channel is at least as important as the direct channel

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